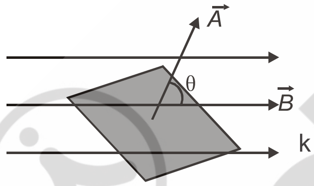
Chapter 13

Electromagnetic Induction

MAGNETIC FLUX

$\phi = \vec{B}.\vec{A} = BA\cos\theta$ (θ = Angle between $\vec{B}$ and $\vec{A}$)



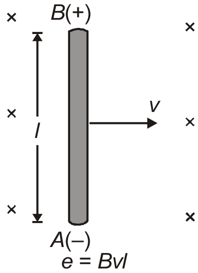
Faraday's Laws of Electromagnetic Induction

$$e = -\frac{d\phi_B}{dt}$$ (for a loop), for a plane coil having N turns

$$e = -\frac{Nd\phi_B}{dt} = -\frac{d(N\phi_B)}{dt}$$

Note : *Negative sign indicates opposition (explained by Lenz's law).*

Induced EMF in different cases

1.


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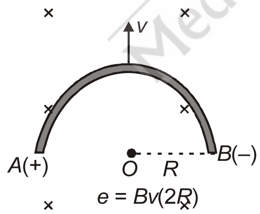

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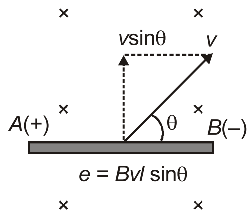

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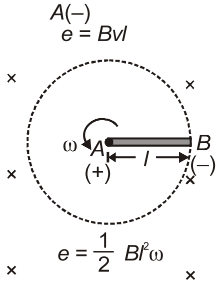

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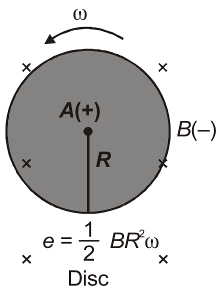

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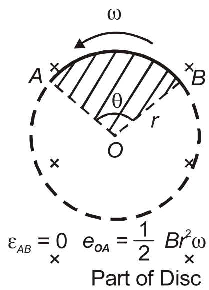

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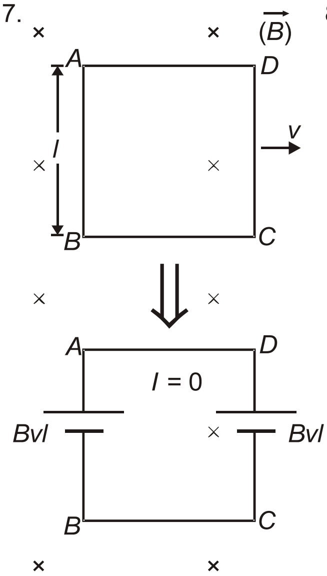

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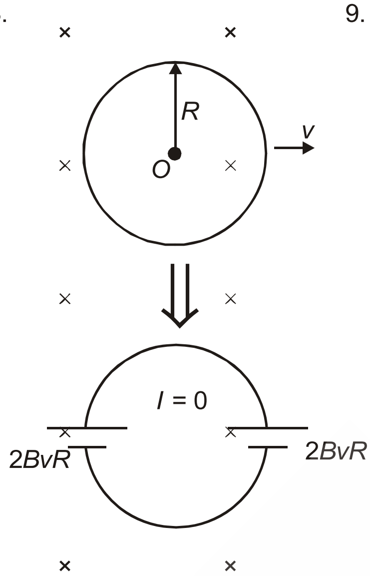

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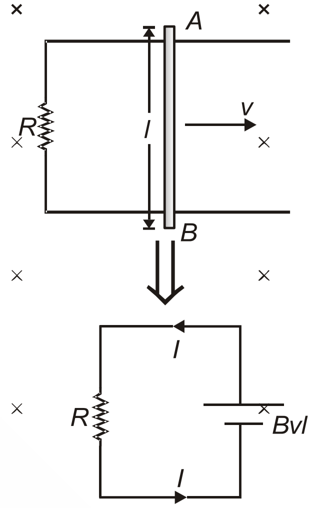
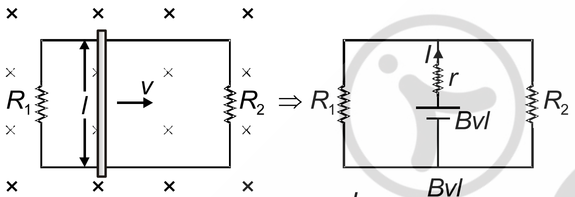


$$I = \frac{Bvl}{R}$$

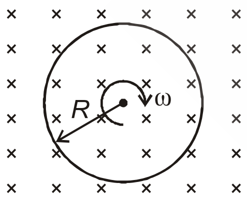
10.



Sliding rod also has a resistance 'r'

$$I = \frac{Bvl}{r + \frac{R_1 R_2}{R_1 + R_2}}$$

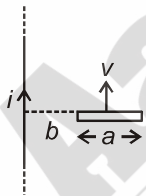
11.



$E_{\text{induced}} = 0$

Ring

12.



$$\varepsilon = \frac{\mu_0 i v}{2\pi} \ln\left(1 + \frac{a}{b}\right)$$

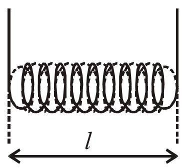
MUTUAL AND SELF INDUCTANCE

Mutual Induction

Important cases :

1. Mutual inductance of two long solenoids :

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$



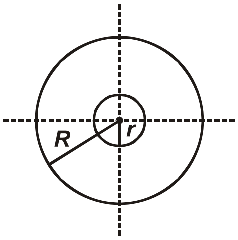
N_1 = Number of turns in one solenoid

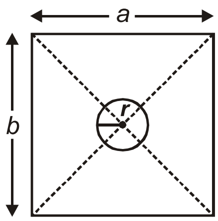
N_2 = Number of turns in other solenoid

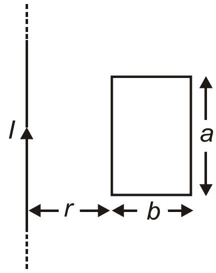
A = Area of cross-section of narrower solenoid

l = Length of solenoid

2. Two loops : $R >> r$







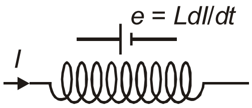
(a) $M = \dfrac{\mu_0 \pi r^2}{2R}$

(b) $M = \dfrac{\mu}{4\pi} \dfrac{8\sqrt{a^2 + b^2}}{ab} \pi r^2$
(For r << a, r << b)

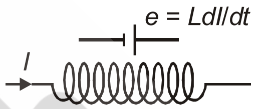
(c) $M = \dfrac{\mu_0}{2\pi} a \log\left(\dfrac{r+b}{r}\right)$

Direction of Induced emf

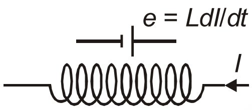
(a) I is increasing



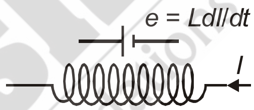
(b) I is decreasing



(c) I is increasing



(d) I is decreasing

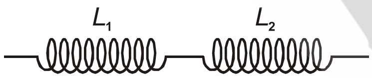


Energy in Inductor

Energy $U_B = \dfrac{1}{2} L I^2$, Energy density = $\dfrac{1}{2} \dfrac{B^2}{\mu_0}$

Combination of Inductors

1. Inductor in series



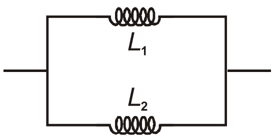
When mutual inductance is neglected $L = L_1 + L_2$

When mutual inductance is considered $L = L_1 + L_2 \pm 2M$

'+' sign is taken when flux of one coil supports the flux in other coil. '–' sign is taken when flux of one coil opposes the flux of other.

Here, $M = K\sqrt{L_1 L_2}$ where K = coefficient of coupling.

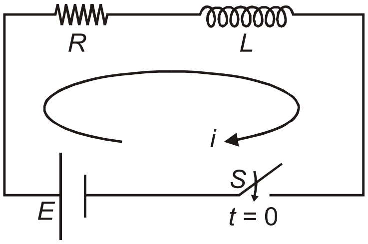
2. Inductor in parallel



$\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ (Neglecting mutual induction)

L-R- CIRCUIT WITH D.C. SOURCE

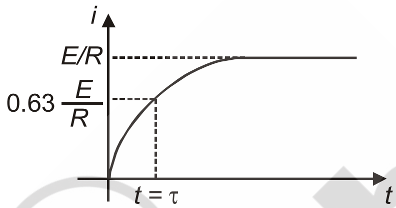
Growth of Current



At t = 0 the switch S is closed

$$i = \frac{E}{R}\left(i - e^{-t/\tau}\right). \text{ Here } \tau = \frac{L}{R}$$

Graph for variation of current (i) with time (t) is shown here.

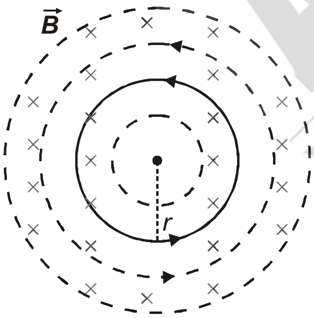


INDUCED ELECTRIC FIELD

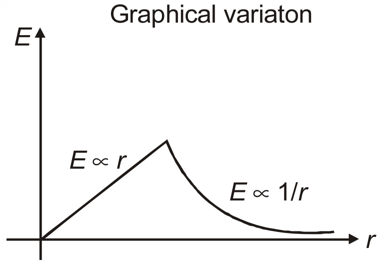
A changing magnetic field produces an electric field. This field is non-conservative and always forms closed loop. Consider a region of magnetic field. The magnetic field strength is increasing at a rate $\frac{d\vec{B}}{dt}$. This creates anticlockwise electric field lines. Electric field strength at distance r from point O is given by

$E = \frac{dB}{dt} \times \frac{r}{2}$ For inside point

$E = \frac{R^2}{2r}\frac{dB}{dt}$ For outside point

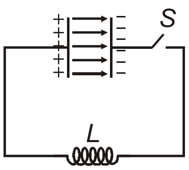


Graphical variaton



LC OSCILLATIONS

A charged capacitor is connected to an inductor and switch is closed at t = 0.

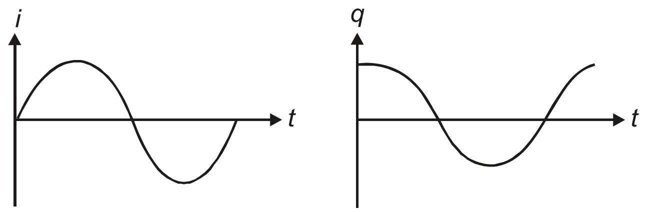


The charge and current vary sinusoidally as,

$q = q_0 \cos \omega t$ [$\because$ at $t = 0$, $q = q_0$]

$i = i_0 \sin \omega t$ [$\because$ at $t = 0$, $i = 0$]

Graphical representation of this variation is as shown.



$i_0 = \frac{q_0}{\sqrt{LC}}, \omega = \frac{1}{\sqrt{LC}}, \nu = \frac{1}{2\pi\sqrt{LC}}$ is frequency of LC oscillations

The maximum charge and maximum current in LC oscillation are related as $\frac{1}{2}LI_{max}^2 = \frac{q_{max}^2}{2C}$

$q_{max} = I_{max}\sqrt{LC}$

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