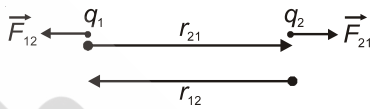
Chapter 10

Electrostatics

Coulomb's Law in Vector Form

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_{12}}|^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_{12}}|^3}\overrightarrow{r_{12}}$$

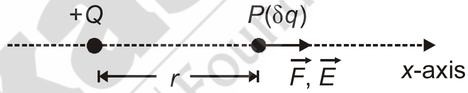
$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_{21}}|^2}\hat{r}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\overrightarrow{r_{21}}|^3}\vec{r}_{21}$$



ELECTRIC FIELD

Electric Field due to a Point Charge (Q) :

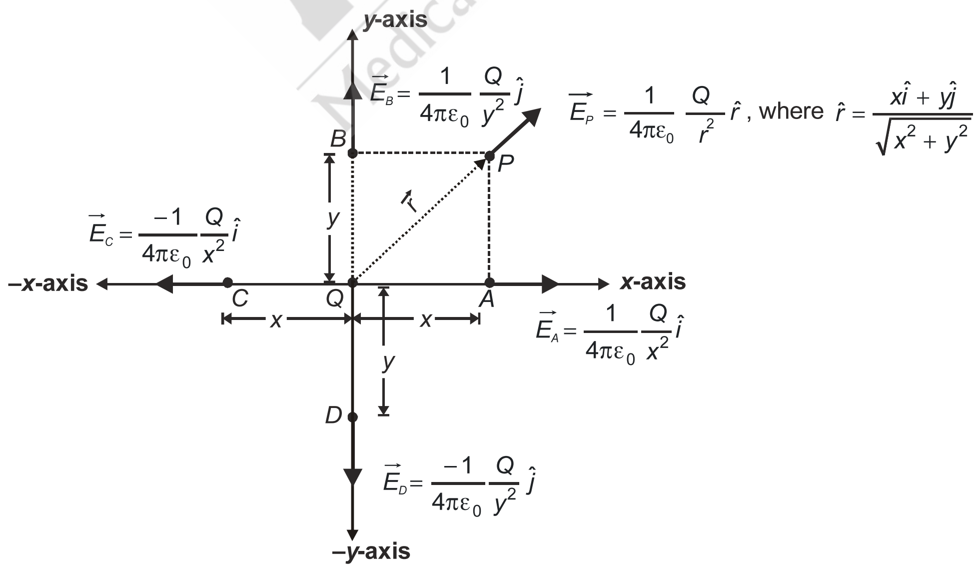
$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q\delta q}{r^2}\hat{i}\ , \quad \vec{E} = \frac{\vec{F}}{\delta q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{i}$$



$$\Rightarrow \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{i}$$

Application

(i) Direction of Electric Field at Various Points (when charge Q is placed at origin) :

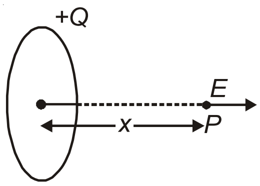


(ii) **Electric field due to a uniformly charged ring on its axis.**

$$E_{axis} = \frac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}}$$

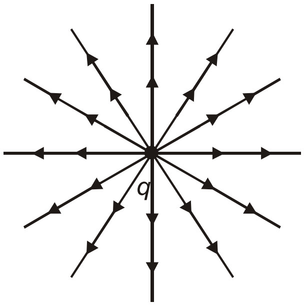
$$E_{centre} = 0$$

At $x = \frac{R}{\sqrt{2}}$, E is maximum.

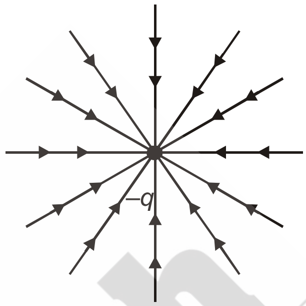


Electric Lines of Force due to Various Configurations

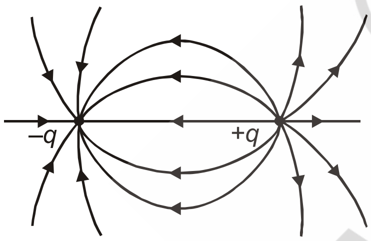
(1) Isolated point charge (+)



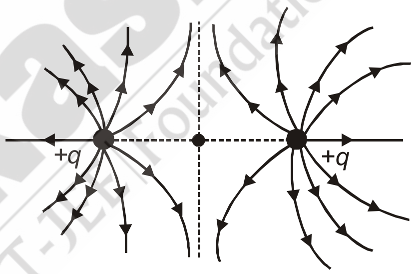
(2) Isolated point Charge (–)



(3) Electric dipole

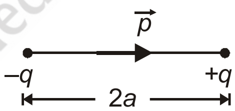


(4) Two identical charges



ELECTRIC DIPOLE

An arrangement of two equal and opposite charges separated by some distance.



Electric Field due to an Electric Dipole

1. For a point P on axial line

$$\vec{E}_{axial} = \frac{2\vec{p}r}{4\pi\varepsilon_0 (r^2 - a^2)^2}$$

For an ideal dipole ($r >> a \Rightarrow r^2 - a^2 \approx r^2$)

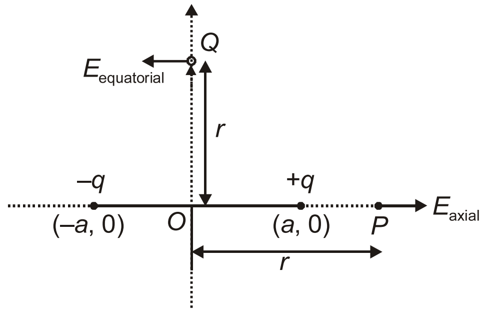
$$\Rightarrow \vec{E}_{axial} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$$

2. For a point Q on equatorial line

$$\vec{E}_{\text{equatorial}} = \frac{-\vec{p}}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}$$

For an ideal dipole ($r >> a \Rightarrow r^2 + a^2 \approx r^2$)

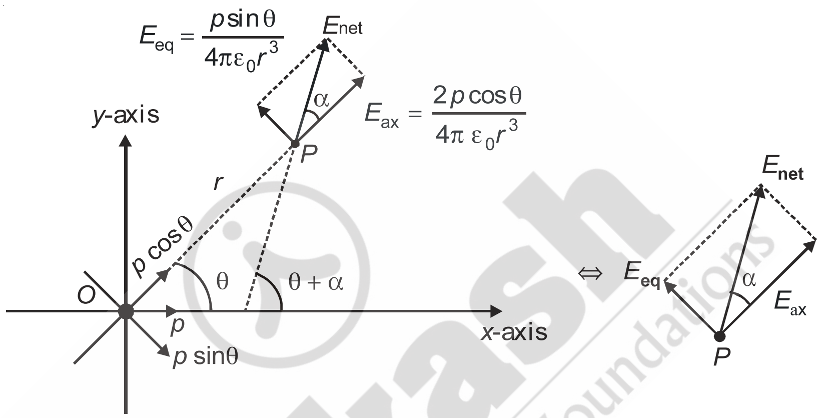
$$\Rightarrow \vec{E}_{\text{equatorial}} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$



3. For an ideal dipole $\vec{E}_{\text{equatorial}} = \frac{-\vec{E}_{\text{axial}}}{2}$ (For same distance from centre of dipole).

4. Electric field at any point in the plane of a **short dipole**

P is a point in x-y plane at a distance r from the centre of dipole, such that OP makes an angle θ with dipole moment.



(a) $E_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$

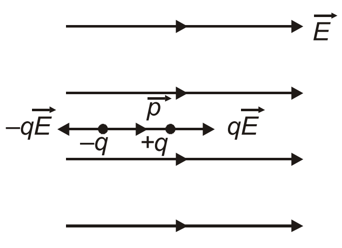
(b) $\tan\alpha = \frac{E_{eq}}{E_{ax}} = \frac{1}{2}\tan\theta \quad \Rightarrow \quad \tan\alpha = \frac{1}{2}\tan\theta$

(c) The net electric field makes an angle $\theta + \alpha$ with dipole moment.

(d) When $\vec{E} \perp \vec{p} \Rightarrow \theta + \alpha = 90° \Rightarrow \theta = \tan^{-1}\sqrt{2}$

Electric Dipole Placed in a Uniform Electric Field (Torque on dipole in uniform electric field)

Case-1 : $\vec{p} \parallel \vec{E}$

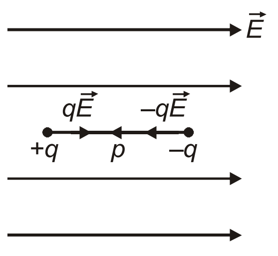


(a) Net force = $q\vec{E} - q\vec{E} = 0$

(b) Net torque = Zero

(c) Stable equilibrium

Case-2 : $\vec{p} \parallel (-\vec{E})$

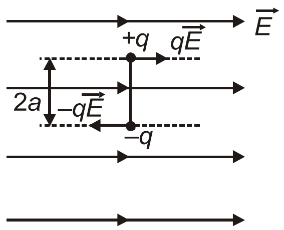


(a) Net force = $q\vec{E} - q\vec{E} = 0$

(b) Net torque = Zero

(c) Unstable equilibrium

Case-3 : $\vec{p} \perp \vec{E}$



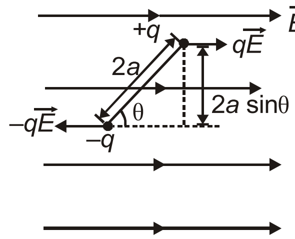
(a) Net force = Zero

(b) $\tau = qE \times 2a = pE$ (This is the maximum value)

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$

(c) Translational equilibrium but not in rotational equilibrium.

Case-4 : $\vec{p}$ makes an angle θ with $\vec{E}$



(a) Net force = Zero

(b) $\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE \sin\theta$

(c) Translational equilibrium but not in rotational equilibrium.

Potential Energy of Dipole

1. The external work required to change the orientation from θ_1 to θ_2 is

   $W_{ext} = -pE[\cos\theta_2 - \cos\theta_1]$
2. Change in potential energy of dipole is

   $U_2 - U_1 = -pE[\cos\theta_2 - \cos\theta_1]$
3. Potential energy of dipole is

   $U = -pE\cos\theta = -\vec{p}.\vec{E}$

ELECTRIC FLUX

The number of field lines that pass through a surface is directly proportional to flux of electric field through that area.

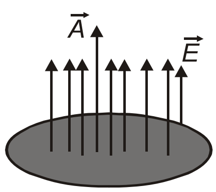
Mathematically, $\phi = \vec{E} \cdot \vec{A}$ (If $\vec{E}$ is uniform and the surface in planar.)

In general, $\phi = \int \vec{E} \cdot d\vec{A}$

Units : $\dfrac{\text{N m}^2}{\text{C}}$ or V-m

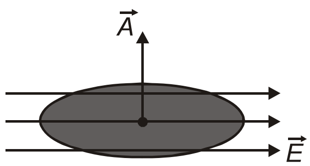
Important cases :

(1) $\vec{E} \,||\, \vec{A}$



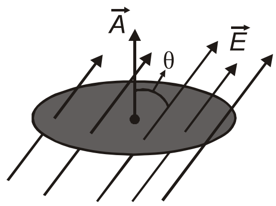
$\phi = EA$

(2) $\vec{E} \perp \vec{A}$



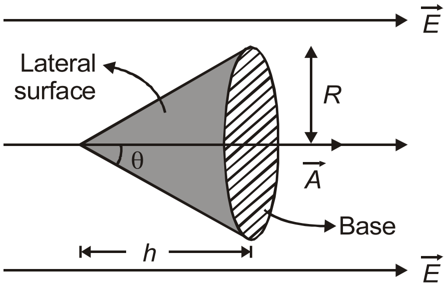
$\phi = 0$

(3) $\vec{E}$ and $\vec{A}$ make an angle θ



$\phi = EA\cos\theta$

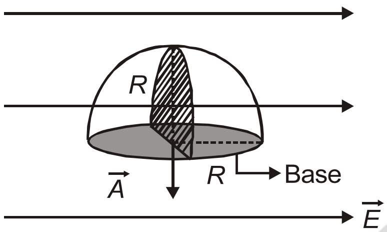
(4)



$\phi_{base} = \vec{E}\,.\,\vec{A} = E \times \pi R^2$

$\phi_{lateral} = -E \times \pi R^2$ ($\because$ Field lines enter through curved surface)

(5)



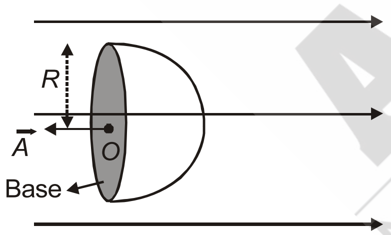
$\phi_{base} = 0$

$\phi_{curved} = 0$ (Total flux that enters = Total flux that leave)

$\phi_{entered} = -E \times \left(\frac{1}{2}\pi R^2\right)$

$\phi_{leaving} = E \times \frac{\pi R^2}{2}$

(6)



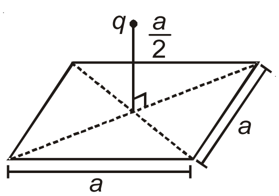
$\phi_{base} = -E \times \pi R^2$

$\phi_{curved} = +E \times \pi R^2$

Electric Flux

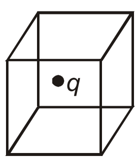
Some frequently asked cases :

1.



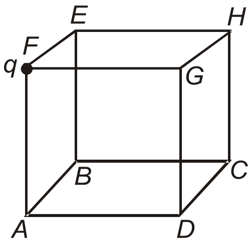
$\phi_{square} = \frac{q}{6\varepsilon_0}$

2.



$\phi_{cube} = \frac{q}{\varepsilon_0}$

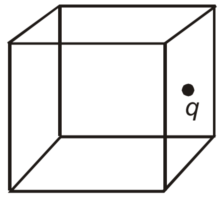
$\phi_{each\ face} = \frac{q}{6\varepsilon_0}$

3. 

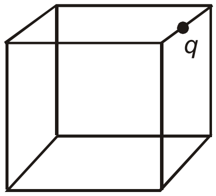
$$\phi_{ABCD} = \frac{q}{24\varepsilon_0} = \phi_{DCHG} = \phi_{BCHE}$$

$$\phi_{ABEF} = 0 = \phi_{ADGP} = \phi_{FGHE}$$

$$\phi_{cube} = \frac{q}{8\varepsilon_0}$$

4. 

$$\phi_{\text{cube}} = \frac{q}{2\varepsilon_0}$$

5. 

$$\phi_{\text{cube}} = \frac{q}{4\varepsilon_0}$$

Important results for fields due to different bodies (derived by Gauss Law)

1. Point charge $Q : \dfrac{kQ}{r^2}$.
2. Shell of charge with charge Q and radius $R : \dfrac{kQ}{r^2}$ (outside) and zero (inside).
3. Sphere of charge with charge Q and radius $R : \dfrac{kQr}{R^3}$ (inside) and $\dfrac{kQ}{r^2}$ (outside).
4. Infinite line of charge with linear charge density $\lambda : \dfrac{2k\lambda}{r}$ (perpendicular to line charge).
5. Infinite plane surface of charge with charge density $\sigma : \dfrac{\sigma}{2\varepsilon_0}$.
6. Infinite conducting sheet of charge with charge density $\sigma : \dfrac{\sigma}{\varepsilon_0}$.

Electric Potential Difference (ΔV)

1. It is the work done against electric field in moving a unit positive charge from one point to other. That is

   $$V_2 - V_1 = -\int_1^2 \vec{E}.d\vec{r}.$$

2. ΔV for two points at a distance r_1 and r_2 from a point charge Q

   $$V_2 - V_1 = \Delta V = KQ\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

3. Change in potential energy of a charge q when moved across ΔV is $\Delta U = q.\Delta V$.
4. ΔV between two points in electric field does not depend on path.

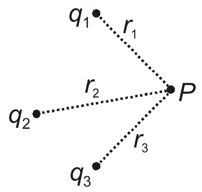
ELECTRIC POTENTIAL (V)

1. V at a point is work done against electric field in moving a unit positive test charge from infinity to that point is $V = -\int_\infty^r \vec{E}.\vec{dr}$.

2. Potential due to a point charge Q at a distance r is $V = \dfrac{KQ}{r}$.

3. Potential due to a dipole at distance r at angle θ is $V = \dfrac{Kp\cos\theta}{r^2}$

4. Potential due to system of point charges is $V_P = \left(\dfrac{Kq_1}{r_1} + \dfrac{Kq_2}{r_2} + \dfrac{Kq_3}{r_3}\right)$.



If V and E are functions of x, then $V_2 - V_1 = -\int_{x_1}^{x_2} E\,dx$.

In general, $V_2 - V_1 = \int_{x_1}^{x_2} E_x dx - \int_{y_1}^{y_2} E_y dy - \int_{z_1}^{z_2} E_z dz$

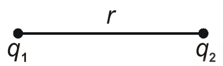
Relation between Electric Field and Potential

1. $E_x = -\dfrac{\delta V}{\delta x}$, $E_y = -\dfrac{\delta V}{\delta y}$, $E_z = -\dfrac{\delta V}{\delta z}$.

2. If V is a function of single variable r, $E = -\dfrac{dV}{dr}$.

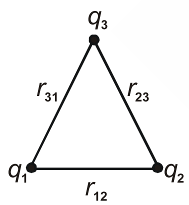
Electric Potential Energy

1. For a two point charge system



$$U = \frac{Kq_1q_2}{r}$$

2. For a three point charge system



$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right]$$

CAPACITOR

It is a device used to store electric energy in the form of electric field.

CAPACITANCE

Capacitance of a conductor is measure of ability of conductor to store electric charge and hence electric energy on it.

When charge is given to a conductor its potential increases. It is found that

$V \propto Q$

or, $Q \propto V$

$Q = CV$

where C is the capacitance and its unit is farad (F).

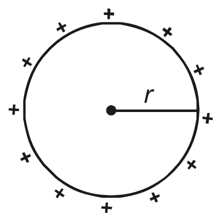
***C* depends on**

- Shape and size of conductors and their relative placement w.r.t. each other.
- Medium surrounding the conductor.

Capacitance of Isolated Spherical Conductor

$C = 4\pi\varepsilon_0 r$

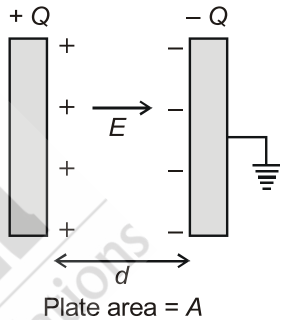
Capacitance of Earth $C_e = 4\pi\varepsilon_0 R_e = 711\ \mu F$



Capacitance of a Parallel Plate Capacitor

1. Electric field in between plates

$$E = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$



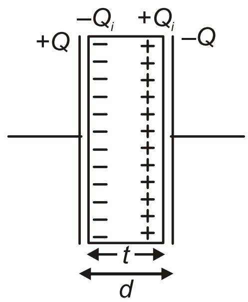
2. Potential difference between the plates = $\dfrac{Qd}{A\varepsilon_0} = \dfrac{\sigma d}{\varepsilon_0}$

3. Capacitance = $\dfrac{\varepsilon_0 A}{d}$

4. Force of attraction between the plates = $\dfrac{Q^2}{2A\varepsilon_0} = \dfrac{\sigma^2 A}{2\varepsilon_0} = \dfrac{QE}{2}$

Parallel Plate Capacitor with Dielectric Slab

(a) Induced charge $Q_i = -Q\left(1-\dfrac{1}{K}\right)$, K is dielectric constant.



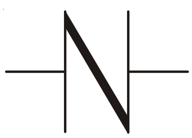
(b) Capacitance, $C = \dfrac{\varepsilon_0 A}{(d-t)+\dfrac{t}{K}}$.

(c) For conducting slab, $K = \infty$

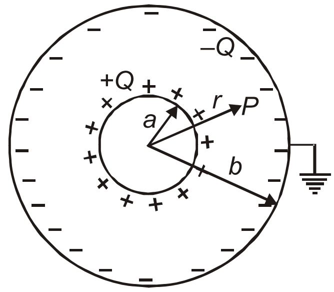
$\Rightarrow Q_i = -Q$ and $C = \dfrac{\varepsilon_0 A}{d-t}$

(d) The capacitance of a parallel plate capacitor is C. If its plates are connected by an inclined conducting rod, the new capacitance is infinity.

$\boxed{C = \infty}$



Spherical Capacitor



1. Potential difference between plates

$$V = KQ\left[\frac{b-a}{ba}\right]$$

2. Electric field at any point P between plates

$$E = \frac{KQ}{r^2}$$

3. Potential at any point P between plates

$$V = \frac{KQ}{r} - \frac{KQ}{b}$$

4. Capacitance $C = \dfrac{4\pi\varepsilon_0 ab}{b-a}$

5. If the inner surface is grounded, capacitance $C = \dfrac{4\pi\varepsilon_0 b^2}{b-a}$

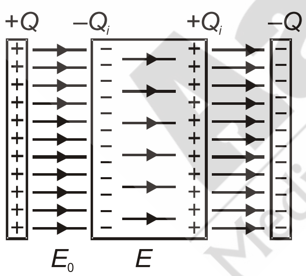
Cylindrical Capacitance of a Long Capacitor

Potential difference between plates

$$V = \frac{2KQ}{l} ln\left(\frac{b}{a}\right)$$

Dielectric Polarisation

When a dielectric slab is placed between the plates of capacitor its polarisation take place. Thus a charge $-Q_i$, appear on its left face and $+Q_i$ appears on its right face.

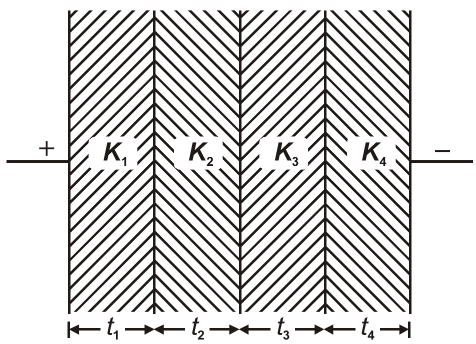


$$Q_i = Q\left(1 - \frac{1}{k}\right)$$

$$E_0 = \frac{Q}{A\varepsilon_0};\ E = \frac{Q}{A\varepsilon_0 k} = \frac{E_0}{k}$$

Effective Capacitance in Some Important Cases

1. $C = \dfrac{\varepsilon_0 A}{\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3} + \dfrac{t_4}{K_4}}$

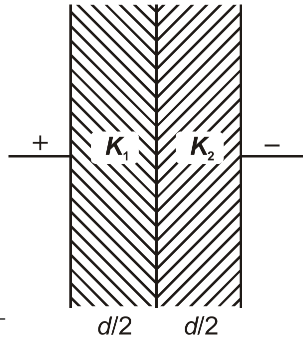


For two dielectrics,

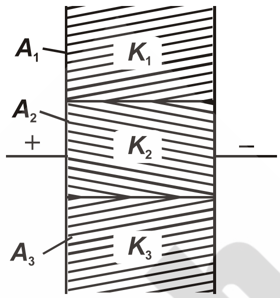
If $t_1 = t_2 = \dfrac{d}{2}$

$$C = \frac{\varepsilon_0 A}{\dfrac{d}{2K_1} + \dfrac{d}{2K_2}} = \frac{2\varepsilon_0 A}{d\left(\dfrac{1}{K_1} + \dfrac{1}{K_2}\right)}$$

$$\Rightarrow \quad C = \left(\frac{2K_1K_2}{K_1 + K_2}\right)\frac{\varepsilon_0 A}{d} \quad K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$



2. $C = \dfrac{\varepsilon_0 [K_1A_1 + K_2A_2 + K_3A_3]}{d}$

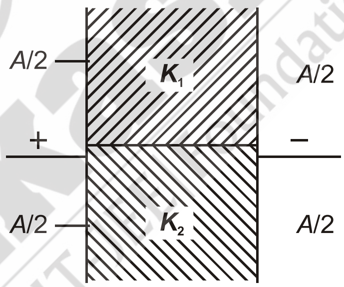


For two dielectrics,

If $A_1 = A_2 = \dfrac{A}{2}$

$$\Rightarrow C = \frac{\varepsilon_0\left(K_1\dfrac{A}{2} + K_2\dfrac{A}{2}\right)}{d}$$

$$\Rightarrow C = \left(\frac{K_1 + K_2}{2}\right)\frac{\varepsilon_0 A}{d} \quad K_{eq} = \frac{K_1 + K_2}{2}$$



COMBINATION OF CAPACITORS

1. **Capacitors in Series** (three capacitors)

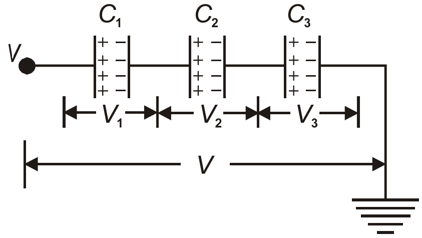
$V_1 = \dfrac{Q}{C_1}$, $V_2 = \dfrac{Q}{C_2}$ and $V_3 = \dfrac{Q}{C_3}$

$V = V_1 + V_2 + V_3$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

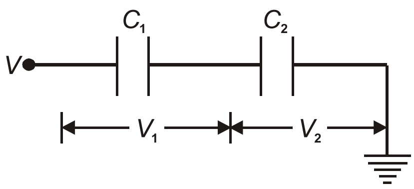
$$V = \frac{Q}{C_{eq}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



2. Two Capacitors in Series

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2}$$



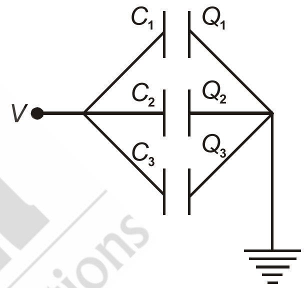
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} \qquad Q = C_{eq}V$$

Potential Dividing Rule

$$V_1 = \frac{C_2}{C_1 + C_2}V \quad V_2 = \frac{C_1}{C_1 + C_2}V$$

3. Capacitors in Parallel



$Q_1 = C_1V,\ Q_2 = C_2V, \quad Q_3 = C_3V$

$\Rightarrow Q = C_1V + C_2V + C_3V$

$\Rightarrow Q = (C_1 + C_2 + C_3)V$ and $Q = C_{eq}V$

$C_{eq} = C_1 + C_2 + C_3$

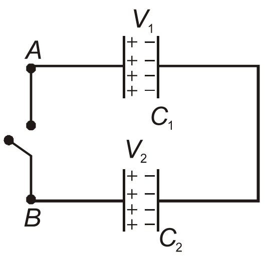
Energy Stored in a Capacitor

Energy stored in a capacitor of capacitance C, charge Q and potential difference V across it is given by

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$$

Sharing of Charge

Case-1 : Two capacitors charged to potentials V_1 and V_2 are connected end to end as shown

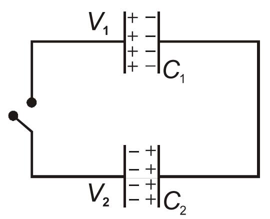


(a) Final common potential $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

(b) Charge flown through key $= \dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)$ in the direction A to B.

(c) Loss of energy $= \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Case-2 : If positive terminal is connected to negative terminal

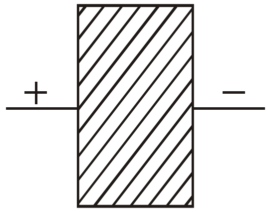


(a) Final common potential $V = \dfrac{C_1V_1 - C_2V_2}{C_1 + C_2}$

(b) Loss of energy $= \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 + V_2)^2$

Inserting a Dielectric Slab

1 When battery is disconnected (isolated) (charge is constant)



Q_0 = initial charge

C_0 = initial capacitance

V_0 = initial potential

E_0 = initial energy

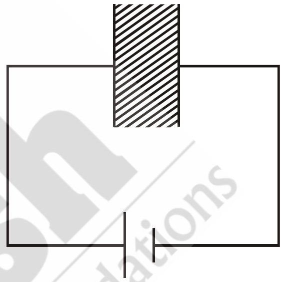
(a) New capacitance = KC_0

(b) New potential difference = $\dfrac{Q_0}{KC_0} = \dfrac{V_0}{K}$

(c) New energy stored = $\dfrac{1}{2}(KC_0)\left(\dfrac{V_0}{K}\right)^2 = \dfrac{E_0}{K}$

(d) Note that charge on each plate remains same.

2. When battery is connected (voltage is constant)



(a) $C = KC_0$

(b) $V = V_0$

(c) $Q = KQ_0$

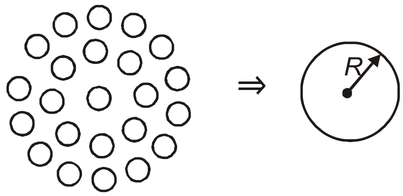
(d) $E = \dfrac{1}{2}(KC_0)(V_0)^2 = KE_0$

Combining Charged Drops

When n droplets of radius r_0 having equal charge Q_0 combined to form a bigger drop of radius R.

(a) $n\dfrac{4}{3}\pi r_0^{\,3} = \dfrac{4}{3}\pi R^3$

$\Rightarrow R = n^{1/3}r_0$



(b) $C = n^{1/3}C_0$

(c) Total charge = nQ_0

(d) $V = \dfrac{nQ_0}{C} = \dfrac{nQ_0}{n^{1/3}C_0} = n^{2/3}V_0$

(e) Total energy = $\dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{(nQ_0)^2}{2n^{1/3}C_0} = n^{5/3}\,U_0$

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