Chapter 6

Gravitation

VARIATION IN THE VALUE OF *g*

1. At height h (Above earth's surface)

$$g' = \frac{g}{\left[1 + \frac{h}{R_e}\right]^2}$$

If $h = R_e$, $g' = \frac{g}{4}$

If $h << R_e$ then $g' = g\left(1 - \frac{2h}{R_e}\right)$

2. At depth *"x"* (below earth's surface)

$g' = g\left\{1 - \frac{x}{R_e}\right\}$, at the centre of earth $g' = 0$, weight = 0

3. **Due to Rotation of Earth :**

Apparent value of acceleration due to gravity.

$g' = g - R_e\omega^2\cos^2\lambda$

$\lambda \rightarrow$ angle of latitude

GRAVITATIONAL FIELD INTENSITY AND POTENTIAL (*V*)

1. Gravitational field intensity

$$\vec{I} = \frac{-GM}{r^2}\hat{r}$$

2. Gravitational potential

$$V = -\frac{W}{m} \Rightarrow V = \frac{-GM}{r} \text{ (units J/kg)}$$

Variation of Intensity and Potential

1. For a spherical shell of mass M and radius R

Case-I : $r < R$ (internal point)

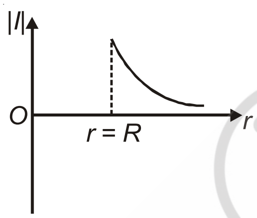
$$I_i = 0,\ V_i = -\frac{GM}{R}$$

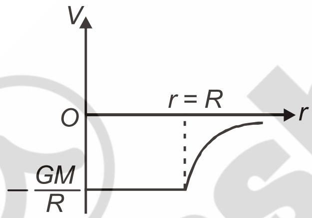
Case-II : $r = R$ (on the surface)

$$I_s = \frac{GM}{R^2},\ V_s = \frac{-GM}{R}$$

Case-III : $r > R$ (outside the shell)

$$I_o = \frac{GM}{r^2},\ V_o = \frac{-GM}{r}$$





2. For Uniform solid Sphere

Case-I : $r < R$ (internal point)

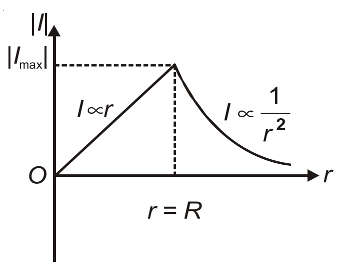
$$I_i = \frac{GMr}{R^3},\ V_i = \frac{-GM}{2R^3}(3R^2 - r^2)\ \text{At centre}\ V_c = -\frac{3GM}{2R} = \frac{3}{2}V_{\text{Surface}}$$

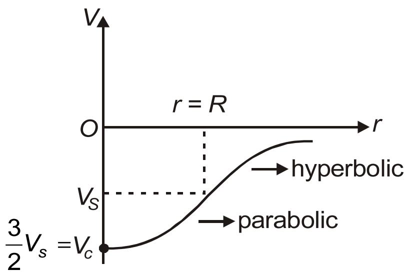
Case-II : $r = R$ (on the surface)

$$I_s = \frac{GM}{R^2},\ \ V_s = \frac{-GM}{R}$$

Case-III : $r > R$ (outside the surface)

$$I_o = \frac{GM}{r^2},\ \ V_o = \frac{-GM}{r}$$

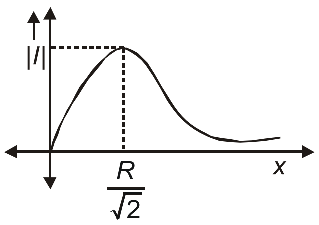


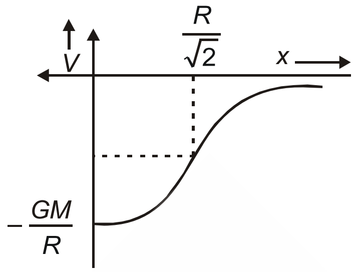


3. Gravitational intensity and potential on the axis of uniform ring of mass M radius R at distance x from centre.

$$I = \frac{GMx}{(R^2 + x^2)^{3/2}}, V = \frac{-GM}{\sqrt{R^2 + x^2}}$$

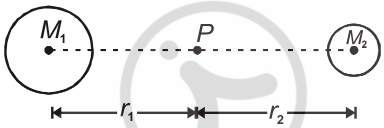
At centre $I = 0$, I is maximum at $x = \pm\frac{R}{\sqrt{2}}$; $I_{max} = \frac{2GM}{3\sqrt{3}R^2}$





4. **Neutral point :** The point P at which gravitational field is zero between two massive bodies, is called neutral point.

$$\boxed{\frac{M_1}{r_1^2} = \frac{M_2}{r_2^2}}$$



GRAVITATIONAL POTENTIAL ENERGY

At earth surface $U = -\frac{GM_e m}{R_e}$, At height h, $U = \frac{-GM_e m}{R_e + h}$

Energy required to escape = Escape energy = + $\frac{GM_e m}{R_e}$ = Binding energy.

ESCAPE VELOCITY

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{8}{3}\pi G R_e{}^2 \rho} = \sqrt{2gR_e}$$

At earth surface, v_e = 11.2 km/s

KEPLER'S LAWS

(1) All planets revolve around the Sun in elliptical orbit having the Sun at one focus.

If e = eccentricity of ellipse then distance of the planet from the Sun at perigee is

$r_p = (1 - e)a$

and distance of the planet from the Sun at apogee is

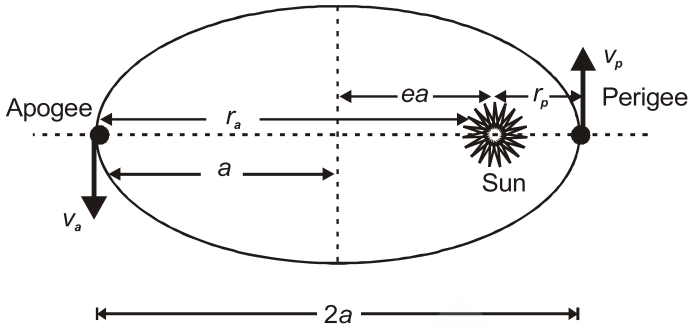
$r_a = (1 + e)a$ (a = semi major axis)

Ratio of orbital speeds at apogee and perigee is

$$\frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{1-e}{1+e}$$

Ratio of angular velocities at apogee and perigee is

$$\frac{\omega_a}{\omega_p} = \left(\frac{r_p}{r_a}\right)^2 = \left(\frac{1-e}{1+e}\right)^2$$



(2) A planet sweeps out equal area in equal time interval *i.e.,* Areal speed of the planet is constant

$$\frac{dA}{dt} = \frac{1}{2}vr = \frac{L}{2m}$$ = constant (L represents angular momentum of planet about the Sun)

(3) Square of time period is proportional to cube of semi-major axis of the elliptical orbit of the planet. *i.e.*, $T^2 \propto a^3$

SATELLITES

Important results regarding satellite motion in circular orbit.

1. Orbital Velocity (v_0) : Gravitational attraction of planet gives necessary centripetal force.

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM}{r}}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR_e^2}{R_e + h}}$$ (h = height above the surface of earth)

$$\Rightarrow \quad v_e = \sqrt{2}v_0$$

2. Time Period : The period of revolution of a satellite is

$$T = \frac{2\pi r}{v_0} = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

For a satellite orbiting close to the earth's surface ($r \simeq R_e$), the time period is minimum and is given by

$$T_{\min} = 2\pi\sqrt{\frac{R_e^3}{GM}} = 2\pi\sqrt{\frac{R_e}{g}}$$

For earth R_e = 6400 km,

g = 9.8 m/s^2

$T_{\min}$ = 84.6 min.= 1.4 h

Thus, for any satellite orbiting around the earth, its time period must be more than $2\pi\sqrt{\frac{R}{g}}$ or 84.6 minutes.

3. Potential Energy (*U*), Kinetic Energy (*K*) and Total Energy (*E*) of satellite:

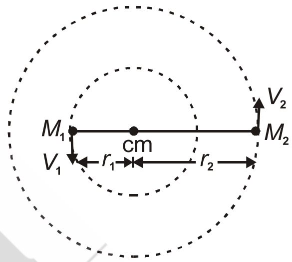
$$U = -\frac{GMm}{r}$$

$$K = \frac{GMm}{2r}$$

$$E = -\frac{GMm}{2r} \quad [K = -E \;\&\; U = 2E]$$

BINARY STAR SYSTEM

Two stars of mass M_1 and M_2 form a stable system when they move in circular orbit about their centre of mass, under their mutual gravitational attraction.



(1) $F = \frac{GM_1M_2}{r^2}$, where r is distance between them (*i.e.,* $r = r_1 + r_2$)

(2) $M_1r_1 = M_2r_2$

(3) $\frac{GM_1M_2}{r^2} = \frac{M_1V_1^2}{r_1}$

(4) $\frac{GM_1M_2}{r^2} = \frac{M_2V_2^2}{r_2^2}$

(5) $r_1 = \frac{M_2r}{M_1 + M_2}$, $r_2 = \frac{M_1r}{M_1 + M_2}$

$$\therefore \quad V_1 = M_2\sqrt{\frac{G}{(M_1 + M_2)r}},$$

$$V_2 = M_1\sqrt{\frac{G}{(M_1 + M_2)r}}$$

when $M_1 = M_2$

$$V_1 = V_2 = \sqrt{\frac{GM}{2r}}$$

$$r_1 = r_2 = \frac{r}{2}$$

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