MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $1^3 - 2^3 + 3^3 - 4^3 + ... + 15^3$ is equal to

(1) 1856 (2) 1854

(3) 1786 (4) 1826

Answer (1)

Sol. $1^3 - 2^3 + 3^3 - 4^3 + ... + 15^3 + (2^3 + 4^3 + ... + 14^3) - (2^3 + 4^3 + ... + 14^3)$

$= (1^3 + 2^3 + 3^3 + ... + 15^3) - 2(2^3 + 4^3 + ... + 14^3)$

$$= \left(\frac{15\times 16}{2}\right)^2 - 2\times 2^3\left(1^3 + 2^3 + ... + 7^3\right)$$

$$= (15\times 8)^2 - 16\left(\left(\frac{7\times 8}{2}\right)^2\right)$$

= 1856

2. $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{32\cos^4\theta}{1+e^{\sin\theta}}d\theta$ is equal to

(1) $3\pi + 8$ (2) $3\pi + 4$

(3) $4\pi + 8$ (4) $4\pi + 4$

Answer (1)

Sol. $$I = \int_{-\pi/4}^{\pi/4} \frac{32\cos^4\theta}{1+e^{\sin\theta}}d\theta$$

$$I = \int_0^{\pi/4}\left(\frac{32\cos^4\theta}{1+e^{\sin\theta}} + \frac{32\cos^4(-\theta)}{1+e^{\sin(-\theta)}}\right)d\theta$$

$$= \int_0^{\pi/4}\left(\frac{32\cos^4\theta}{1+e^{\sin\theta}} + \frac{32\cos^4\theta}{1+e^{-\sin\theta}}\right)d\theta$$

$$= 32\int_0^{\pi/4}\cos^4\theta\left(\frac{1}{1+e^{\sin\theta}} + \frac{1}{1+e^{-\sin\theta}}\right)d\theta$$

$$= 32\int_0^{\pi/4}\cos^4\theta\, d\theta$$

$$= 32\int_0^{\pi/4}\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

$$= \frac{32}{4}\int_0^{\pi/4}(1 + 2\cos 2\theta + \cos^2 2\theta)d\theta$$

$$= \frac{32}{4}\int_0^{\pi/4}\left(1 + 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right)d\theta$$

$$= \frac{32}{8}\int_0^{\pi/4}(3 + 4\cos 2\theta + \cos 4\theta)d\theta$$

$$= 4\left[3\theta + 2\sin 2\theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi/4}$$

$$= 3\pi + 8$$

3. If $x_1, x_2, ..., x_{25}$ be 25 observations such that $\sum_{i=1}^{25}(x_i+5)^2 = 2500$ and $\sum_{i=1}^{25}(x_i-5)^2 = 1000$. Then, the ratio of Mean and Standard deviation of the given observations is

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) $\frac{2}{3}$ (4) $\frac{1}{5}$

Answer (2)

Sol. $$\sum_{i=1}^{25}(x_i+5)^2 = 2500$$

$$\sum_{i=1}^{25}x_i^2 + 10\sum_{i=1}^{25}x_i + 25\times 25 = 2500$$

$$\sum_{i=1}^{25}x_i^2 + 10\sum_{i=1}^{25}x_i = 1875 \quad ...(1)$$

$$\sum_{i=1}^{25}(x_i-5)^2=1000$$

$$\sum_{i=1}^{25}x_i^2-10\sum_{i=1}^{25}x_i=375 \qquad \ldots(2)$$

$$2\sum_{i=1}^{25}x_i^2=2250$$

$$\sum_{i=1}^{25}x_i^2=1125$$

and $\sum_{i=1}^{25}x_i=75$

Mean = $\dfrac{\sum_{i=1}^{25}x_i}{25}=\dfrac{75}{25}=3$

S.D = $\sqrt{\dfrac{\sum_{i=1}^{25}x_i^2}{25}-(\text{Mean})^2}$

$=\sqrt{36}=6$

$\dfrac{\text{Mean}}{S.D}=\dfrac{3}{6}=\dfrac{1}{2}$

4. In the expansion of $(1+\alpha x)^{26}$ and $(1-\alpha x)^{28}$, the coefficient of middle term is same, then the value of α is

(1) $\dfrac{7}{22}$ (2) $\dfrac{7}{27}$

(3) $\dfrac{5}{27}$ (4) $\dfrac{5}{22}$

Answer (2)

Sol. $(1+\alpha x)^{26}$

Middle term position $=\dfrac{26}{2}+1$ = 14th term.

$T_{14}={}^{26}C_{13}(\alpha x)^{13}={}^{26}C_{13}\alpha^{13}x^{13}$

$(1-\alpha x)^{28}$

Middle term : $\dfrac{28}{2}+1$ = 15th term

$T_{15}={}^{28}C_{14}(-\alpha x)^{14}={}^{28}C_{14}\alpha^{14}x^{14}$

Now,

${}^{26}C_{13}\alpha^{13}={}^{28}C_{14}\alpha^{14}$

$\therefore \alpha=\dfrac{{}^{26}C_{13}}{{}^{28}C_{14}}=\dfrac{7}{27}$

5. a_1, a_2, a_3,.....,a_n are in A.P. and sum of first 10 terms is 160. g_1, g_2, g_3,.......g_n are in G.P., were $g_1 + g_2 = 8$. If the first term of A.P. is equal to common ratio of G.P. and first term of G.P. is equal to common difference of A.P., then sum of all possible values of g_1 is equal to

(1) $\dfrac{34}{9}$ (2) $\dfrac{28}{9}$

(3) $\dfrac{23}{3}$ (4) $\dfrac{28}{5}$

Answer (1)

Sol. $g_1 = d$ and $a_1 = r$....(given)

$160=\dfrac{10}{2}(2a_1+9d)$.....(given)

$\Rightarrow 160 = 5(2r + 9d)$

$\Rightarrow 2r + 9d = 32$

$g_2 = g_1 \times r \Rightarrow g_2 = dr \quad (g_1 = d)$

$g_1 + g_2 = 8$ ...... (given)

$\Rightarrow d(1 + r) = 8$

$\Rightarrow d\left(1+\dfrac{32-9d}{2}\right)=8$

$\Rightarrow 9d^2 - 34d + 16 = 0$

sum of all possible value of g_1

= sum of all possible value of d

$=\dfrac{34}{9}$

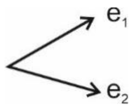
6. Consider e_1 and e_2 be roots of the equation $x^2 - ax + 2 = 0$. Set of values of a for which e_1 and e_2 are eccentricities of hyperbolas then $a \in [\alpha, \beta)$ and set of values of a for which e_1 and e_2 are eccentricity of a hyperbola and an ellipse is (γ, ∞) then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(1) 26 (2) 24

(3) 18 (4) 32

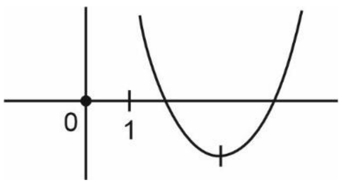
Answer (1)

Sol. $x^2 - ax + 2 = 0$



S. 1 $\Rightarrow$ e_1 and e_2 are eccentricity of hyperbola

$\Rightarrow e_1, e_2 > 1$



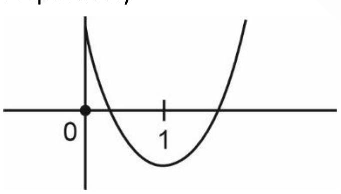
$\Rightarrow f(1) > 0, -\frac{b}{2a} > 1, D \geq 0$ $\quad \Rightarrow a^2 - 8 \geq 0 \Rightarrow |a| \geq 2\sqrt{2}$

$\Rightarrow \frac{a}{2} > 1 \Rightarrow a > 2$

$1 - a + 2 > 0 \Rightarrow a < 3$

$\Rightarrow a \in [2\sqrt{2}, 3)$

S. 2 $\Rightarrow$ e_1 and e_2 be eccentricity of hyperbola & Ellipse respectively



$\Rightarrow f(1) < 0$ and $D > 0$

$\Rightarrow 1 - a + 2 < 0 \quad \Rightarrow a > 3$

$a^2 - 8 > 0 \quad \Rightarrow |a| \geq 2\sqrt{2}$

$\Rightarrow a \in (3, \infty)$

$\Rightarrow \alpha^2 + \beta^2 + r^2 = 8 + 9 + 9 = 26$

7. There are $(n + 1)$ coins. 'n' coins are unbiased coins and one coin has two heads. A coin is randomly chosen and tossed once. If the probability of getting head is $\frac{9}{16}$, then the value of n is

(1) 5 (2) 6

(3) 7 (4) 8

Answer (3)

Sol. There are n unbiased coin, 1 biased coin

E_1 : Biased coin is picked

E_2 : Unbiased coin is picked

H : Output is Head

$$P(H) = P(E_1) \cdot P\left(\frac{H}{E_1}\right) + P(E_2) \cdot P\left(\frac{H}{E_2}\right)$$

$$\frac{9}{16} = \frac{1}{n+1} \times 1 + \frac{n}{n+1} \times \frac{1}{2}$$

$$\Rightarrow \frac{9}{16} = \frac{2+n}{2(n+1)}$$

$\Rightarrow$ $9n + 9 = 16 + 8n$

$\Rightarrow$ $n = 7$

8. The number of 4 letter words which can be made using the letters of the word INCONSEQUINTAL without repetition using 2 vowels and 2 consonants is equal to

(1) 3460 (2) 3600

(3) 4200 (4) 2400

Answer (2)

Sol. Vowels : {A, E, I, O, U}

Consants : {N, C, S, Q, T, L}

Two vowels and two consonants :

$^5C_2 \times {}^6C_2 \times 4!$

$= 10 \times 15 \times 24 = 3600$

9. If $\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$ & $\beta=\frac{1}{3\alpha}$ then the value of 6(α + β) is equal to

(1) 7 (2) 9

(3) 8 (4) 6

Answer (1)

Sol. $\tan^{-1}(1-\alpha)+\tan^{-1}(1-\beta)=\frac{\pi}{4}$

$$\tan^{-1}\left(\frac{1-\alpha+1-\beta}{1-(1-\alpha)(1-\beta)}\right)=\frac{\pi}{4}$$

$$\frac{2-\alpha-\beta}{1-(1-\alpha-\beta+\alpha\beta)}=1$$

$$\frac{2-(\alpha+\beta)}{\alpha+\beta-\frac{1}{3}}=1$$

$$2-(\alpha+\beta)=\alpha+\beta-\frac{1}{3}$$

$$2(\alpha+\beta)=2+\frac{1}{3}=\frac{7}{3}$$

$$\alpha+\beta=\frac{7}{6}$$

$\therefore$ 6(α + β) = 7

10. If $1+\cos x=\sqrt{3}\sin x$ where $x\in(-2\pi,2\pi)$. Then, the sum of all the values of x satisfy the given equation is

(1) 5π (2) $\frac{4\pi}{3}$

(3) 4π (4) $\frac{-4\pi}{3}$

Answer (4)

Sol. $1+\cos x=\sqrt{3}\sin x$

$$\sqrt{3}\sin x-\cos x=1$$

$$\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x=\frac{1}{2}$$

$$\left(\cos\frac{\pi}{6}\right)\sin x-\left(\sin\frac{\pi}{6}\right)\cos x=\frac{1}{2}$$

$$\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$$

$$x-\frac{\pi}{6}=\frac{\pi}{6}+2n\pi$$

Or $x=\frac{\pi}{6}=\frac{5\pi}{6}+2n\pi$

$$x=\pi,-\pi,\frac{-5\pi}{3},\frac{\pi}{3}$$

$$\sum x=\frac{\pi}{3}+\pi-\pi-\frac{5\pi}{3}=\frac{-4\pi}{3}$$

11. Points P and Q lie on the parabola $y^2 = 12x$. The ratio of their y-coordinates is 1 : 2 and the length of line segment PQ is $3\sqrt{13}$. If line PQ makes an angle θ with positive x-axis in anticlockwise direction, θ ∈ (0, π), then sin θ is equal to

(1) $\frac{1}{\sqrt{13}}$ (2) $\frac{3}{\sqrt{13}}$

(3) $\frac{2}{\sqrt{13}}$ (4) $\frac{1}{\sqrt{12}}$

Answer (3)

Sol. $y^2 = 12x$

Let $P(3t_1^2,6t_1)$, $Q(3t_2^2,6t_2)$

$$\frac{6t_1}{6t_2}=\frac{1}{2}$$

$\Rightarrow$ $t_2 = 2t_1$

$$PQ^2=(3t_2^2-3t_1^2)^2+(6t_2-6t_1)^2$$

$$\Rightarrow(3\sqrt{13})^2=9(3t_1^2)^2+36t_1^2$$

$$\Rightarrow 117=81t_1^4+36t_1^2$$

$$\Rightarrow 9t_1^4+4t_1^2-13=0$$

$\Rightarrow (9t_1^2+13)(t_1^2-1)=0$

$\Rightarrow t_1^2=1$

$t_1 = 1,\ t_2 = 2$

$P(3, 6) \quad Q(12, 12) \rightarrow$ Slope: $\tan\theta = \frac{6}{9} = \frac{2}{3}$

$t_1 = -1,\ t_2 = -2$

$P(3, -6) \quad Q(12, -12) \rightarrow$ Slope: $\tan\theta = \frac{-6}{9} = \frac{-2}{3}$

$\sin\theta = \frac{2}{\sqrt{13}}$ in both cases.

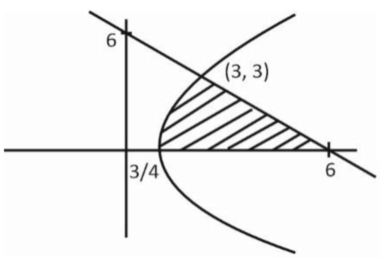
12. If the area (in square units) of the region $\{(x, y) : y \le 6-x,\ y^2 \le 4x-3,\ x \ge 0,\ y \ge 0\}$ is

(1) 15 (2) 9

(3) 8 (4) 7

Answer (2)

Sol.



Point of intersection of $y^2 = 4x-3$ and $y = 6-x$ is (3, 3)

Required area = $\int_0^3 \left[(6-y)-\left(\frac{y^2+3}{4}\right)\right]dy$

$= \left[6y - \frac{y^2}{2} - \frac{y^3}{12} - \frac{3y}{4}\right]_0^3$

$= \frac{21(3)}{4} - \frac{9}{2} - \frac{27}{12}$

$= \frac{36}{4}$

= 9 sq. units

13. The domains of the function $f(x) = \sin^{-1}\left(\frac{x+[x]}{3}\right)$ is $[\alpha, \beta)$. The value of $\alpha^2 + \beta^2$ is equal to

(where [·] represents G.I.F)

(1) 4 (2) 5

(3) 3 (4) 2

Answer (2)

Sol. $\sin^{-1}\left(\frac{x+[x]}{3}\right) \Rightarrow -1 \le \frac{x+[x]}{3} \le 1$

$\Rightarrow -3 \le x + [x] \le 3$

Case-I: for $x \in [-2, -1)$

$[x] = -2$

$x + [x] \in [-4, -3)$

$\Rightarrow$ (no value of x)

Case-II: $x \in [-1, 0)$

$[x] = -1$

$x + [x] \in [-2, -1) \rightarrow$ all value of $x \in [-1, 0)$

Case-III: $x \in [0, 1)$

$[x] = 0$

$x + [x] \in [0, 1) \rightarrow$ all value of $x \in [0, 1)$

Case-IV: $x \in [1, 2)$

$[x] = 1$

$x + [x] \in [2, 3) \rightarrow$ all value of $x \in [1, 2)$

Case-IV: $x \in [2, 3)$

$[x] = 2$

$x + [x] \in [4, 5) \rightarrow$ no value of x

$\therefore x \in [-1, 2)$

$\alpha^2 + \beta^2 = 5$

14. If the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $a, b>0$ with eccentricity e passes through point $P(6,\ 4\sqrt{3})$ and $15(e^2 + 1) = (34)e$ then length of latus rectum of hyperbola $\frac{x^2}{b^2}-\frac{y^2}{2(a^2+1)}=1$ is

(1) 10 (2) 12

(3) 14 (4) 8

Answer (1)

Sol. $(6,\ \sqrt{48})$ lie on hyperbola,

$\Rightarrow \frac{36}{a^2}-\frac{48}{b^2}=1$ ...(1)

and $15(e^2+1)=34e$

$\Rightarrow 15e^2-34e+15=0$

$e=\frac{34\pm\sqrt{34^2-4\cdot 15^2}}{30}$

$=\frac{34\pm\sqrt{34^2-30^2}}{30}=\frac{34\pm\sqrt{64\times 4}}{30}=\frac{34\pm 16}{30}$

Since $e>1 \Rightarrow e=\frac{50}{30}=\frac{5}{3}$

$\Rightarrow e^2=\frac{b^2}{a^2}+1=\frac{25}{9}\Rightarrow\frac{b}{a}=\frac{16}{9}$ ...(2)

$\Rightarrow$ solving (1) & (2)

$a^2=9,\quad b^2=16$

Latus rectum $=2\left(\frac{2(a^2+1)}{b}\right)=\frac{4(10)}{4}=10$

15. Consider the circle $x^2 + y^2 + 2gx + 2fy + 25 = 0$, where $g, f \in z$. Centre lies on $2x - y = 4$ and the area equilateral triangle inscribed within this circle is $27\sqrt{3}$ sq. units. The square of the length of chord whose equation is $x = 1$ is equal to

(1) 45 (2) 40

(3) 80 (4) 90

Answer (3)

Sol. $x^2 + y^2 + 2gx + 2fy + 25 = 0$

centre: $(-g, -f)$ lies on $2x - y = 4$

$-2g + f = 4 \Rightarrow f = 2g + 4$

area $=\frac{3\sqrt{3}}{4}R^2=27\sqrt{3}$

$\Rightarrow R = 6$

$36 = g^2 + f^2 - 25$

$\Rightarrow g^2 + f^2 = 61$

$\Rightarrow g^2 + (2g + 4)^2 = 61$

$\Rightarrow 5g^2 + 16g - 45 = 0$

$\Rightarrow (5g - 9)(g + 5) = 0$

$\Rightarrow g=\frac{9}{5}, -5$

$\therefore g = -5, f = -6$ $\therefore$ centre (5, 6)

Length of chord $= 2\sqrt{R^2-d^2}$

$=2\sqrt{36-4^2}=2\sqrt{20}$

$\therefore L^2 = 80$

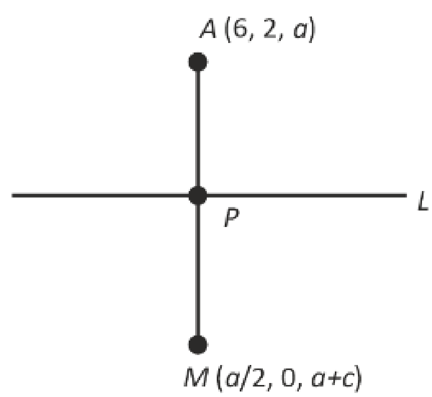
16. If the image of a point $A(6, 2, a)$ with respect to line L : $\frac{x}{1}=\frac{y-1}{2}=\frac{Z-a+1}{b}$ is $\left(\frac{a}{2}, 0, a+c\right)$. Then, the distance of the foot of perpendicular of A on line L from the point (a, b, c) is equal to

(1) $2\sqrt{7}$ (2) $\sqrt{346}$

(3) $\sqrt{421}$ (4) $\sqrt{247}$

Answer (2)

Sol.



$P(\lambda, 2\lambda+1, b\lambda+a-1)$

$\overrightarrow{AP} = (\lambda-6, 2\lambda-1, b\lambda-1)$

$\overrightarrow{AP} \perp L$

$\Rightarrow (\lambda-6)(1)+(2\lambda-1)(2)+b(b\lambda-1)=0$

$\Rightarrow \lambda-6+4\lambda-2+b^2\lambda-b=0$

$\Rightarrow (5+b^2)\lambda = 8+b$

$\Rightarrow \lambda = \dfrac{8+b}{5+b^2}$ ...(1)

$M(2\lambda-6, 4\lambda, 2b\lambda+a-2)$

$2\lambda-6 = \dfrac{a}{2}$

$4\lambda = 0$ and $2b\lambda + a - 2 = a + c$

$\Rightarrow \lambda = 0$ and $a - 2 = a + c$

$c = -2$

from (1)

$\boxed{b=-8}$

$Q(a, b, c) = (-12, -8, -2)$

$P(0, 1, -13)$

$PQ = \sqrt{144+81+121} = \sqrt{346}$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The value of $\lim\limits_{x\to 0} \dfrac{x^2 \sin^2 x}{x^2 - \sin^2 x}$ is equal to

Answer (03)

Sol. $\lim\limits_{x\to 0} \dfrac{x^2\left(x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+...\right)^2}{x^2-\left(x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+...\right)^2}$

$= \lim\limits_{x\to 0} \dfrac{x^4\left(1-\dfrac{x^2}{3!}+....\infty\right)}{x^2-x^2+2\cdot\dfrac{x4}{3!}+...} = \dfrac{3!}{2} = 3$

22. Let matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. If $A^2 + \alpha(adj(adj(A))) + \beta adj(adj(adj(A)))) = \begin{bmatrix} -4 & -4 & 0 \\ -4 & 0 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ then $(\alpha+\beta)^2$ is equal to

Answer (09.00)

Sol. adj $(A) = |A|(A^{-1})$

Here, $|A| = -1$

$M_1 = adj(adj(A)) = |A|^{n-2}(A) = -A$

$M_2 = adj(adj(adj(A))) = |A|^{(n-2)(n-1)}(adjA) = (adjA)$

$\Rightarrow A^2 + \alpha M_1 + \beta M_2 = \begin{bmatrix} 4 & -4 & 0 \\ -4 & 0 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} + \alpha \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} + \beta \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & -4 & 0 \\ -4 & 0 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

Comparing

$\alpha = 2, \beta = 1$

$(\alpha+\beta)^2 = 9$

23.

24.

25.

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