# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer :**

1. If $\alpha = 3 + 4 + 8 + 9 + 13 + \ldots$ upto 40 terms and $(\tan\beta)^{\frac{\alpha}{1020}}$ is the root of the equation $x^2 - x - 2 = 0$. Then, the value of $\sin^2\beta + 3\cos^2\beta$ is

(1) $\frac{5}{2}$ (2) 2

(3) 1 (4) $\frac{1}{2}$

**Answer (3)**

**Sol.** $\alpha = 3 + 4 + 8 + 9 + 13 + \ldots$ upto 40 terms

$= (3 + 8 + 13 + \ldots) + (4 + 9 + \ldots)$

$= \frac{20}{2}[6 + (20 - 1)5] + \frac{20}{2}[8 + (20 - 1)5]$

$= 1010 + 1030 = 2040$

$(\tan\beta)^{\left(\frac{\alpha}{1020}\right)}$ is the root of $x^2 - x - 2 = 0$

$(\tan\beta)^2$ is the root of $x^2 - x - 2 = 0$

$x^2 - x - 2 = 0$

$x = 2, -1$

$(\tan\beta)^2 = 2 \Rightarrow \tan\beta = \pm\sqrt{2}$

$\sin^2\beta + 3\cos^2\beta = \frac{2}{3} + \frac{3}{3} = 1$

2. if $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{9}{16}\right)$, consider statements:

**Statement 1:** $\cos(\alpha + \beta) > 0$

**Statement 2:** $\cos \alpha < 0$

Then which of the following is true?

(1) Statement 1 and 2 are correct

(2) Only statement 1 is correct

(3) Only statement 2 is correct

(4) None of these statements are correct.

**Answer (3)**

**Sol.** $1 > \frac{6}{11} > \frac{1}{2}$

$\Rightarrow \frac{\pi}{2} > \sin^{-1}\left(\frac{6}{11}\right) > \frac{\pi}{6}$

$\Rightarrow 3\sin^{-1}\left(\frac{6}{11}\right) > \frac{\pi}{2}$

$\Rightarrow \cos^{-1}(\alpha) < 0$

also $\frac{9}{16} > \frac{1}{2}$

$\Rightarrow \cos^{-1}\left(\frac{9}{16}\right) < \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$3\cos^{-1}\left(\frac{9}{16}\right) < \pi \Rightarrow \alpha + \beta \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \cos(\alpha + \beta) < 0$

3. A person has 3 different bags & 4 different books. The number of ways in which he can put these books in the bags so that no bag is empty, is

(1) 36 (2) 34

(3) 32 (4) 30

**Answer (1)**

**Sol.** **Inclusion-Exclusion**

Total ways = $3^4$ = 81

Ways with atleast 1 empty bag = ${}^3C_1 \times 2^4 = 48$

Ways with atleast 2 empty bag = ${}^3C_2 \times 1^4 = 3$

$\therefore$ Total ways = 81 – 48 + 3 = 36

4. If $\left(x\sqrt{1-x^2}\right)dy-\left(y\sqrt{1-x^2}-x^2\cos^{-1}x\right)dx=0$ and $\lim\limits_{x\to1^-}y(x)=1$, then $y\left(\frac{1}{2}\right)$ is

(1) $\frac{\pi^2}{36}$

(2) $\frac{\pi}{36}+1$

(3) $\frac{\pi^2}{36}+\frac{1}{2}$

(4) $\frac{\pi}{36}$

**Answer (3)**

**Sol.** $\frac{dy}{dx}-\frac{y}{x}=\frac{-x\cos^{-1}x}{\sqrt{1-x^2}}$

IF $e^{-\int\frac{1}{x}dx}=\frac{1}{x}$

$\therefore\quad \frac{y}{x}=-\int\frac{\cos^{-1}x}{\sqrt{1-x^2}}dx$

let $\cos^{-1}x=t$

$\frac{-1}{\sqrt{1-x^2}}dx=dt$

$\frac{y}{x}=\int t\,dt=\frac{t^2}{2}+c$

$\frac{y}{x}=\frac{(\cos^{-1}x)^2}{2}+c$

$\therefore\quad \lim\limits_{x\to1^-}y(x)=1\Rightarrow\frac{1}{1}=0+c$

$\Rightarrow c=1$

$\therefore\quad y=\frac{x(\cos^{-1}x)^2}{2}+x$

$y\left(\frac{1}{2}\right)=\frac{1}{2}\times\frac{\pi^2}{9\times4}+\frac{1}{2}$

$\boxed{y\left(\frac{1}{2}\right)=\frac{\pi^2}{36}+\frac{1}{2}}$

5. Let $\frac{x^2}{f\left(a^2+2a+7\right)}+\frac{y^2}{f\left(3a+14\right)}=1$ represents an equation of ellipse. The major axis of given ellipse is y-axis and $f$ is a decreasing function. If the range of $a$ is $R-[\alpha,\beta]$ then $\alpha+\beta$ is

(1) 3 (2) 4

(3) 2 (4) 1

**Answer (4)**

**Sol.** $\frac{x^2}{f(a^2+2a+7)}+\frac{y^2}{f(3a+14)}=1$

$f(3a+14)>f(a^2+2a+7)$

$3a+14<a^2+2a+7$

$a^2-a-7>0$

$a\in\left(-\infty,\frac{1-\sqrt{29}}{2}\right)\cup\left(\frac{1+\sqrt{29}}{2},\infty\right)$

$a\in R-\left[\frac{1-\sqrt{29}}{2},\frac{1+\sqrt{29}}{2}\right]$

$\alpha+\beta=\frac{1-\sqrt{29}}{2}+\frac{1+\sqrt{29}}{2}=1$

6. The value of $\int_0^2\frac{\sqrt{x\left(x^2+x+1\right)}}{\sqrt{x+1}\sqrt{x^4+x^2+1}}dx$ is

(1) $\frac{1}{3}\ln\left(2^{3/2}+3\right)$ (2) $\ln\left(2^{3/2}+3\right)$

(3) $\ln\left(3^{3/2}+1\right)$ (4) $\frac{2}{3}\ln\left(2^{3/2}+3\right)$

**Answer (4)**

**Sol.** $\int_0^2\frac{\sqrt{x(x^2+x+1)}}{\sqrt{x+1}\sqrt{x^4+x^2+1}}dx=\int_0^2\frac{\sqrt{x}\sqrt{(x^2+x+1)}}{\sqrt{x+1}\sqrt{x^2+x^2+1}}dx$

$\int_0^2\frac{\sqrt{x}}{\sqrt{x^3+1}}dx$

Let $x^{3/2}=\mu$

$\frac{3}{2}x^{1/2}dx=du$

$$= \frac{2}{3}\int_0^{(2)^{3/2}} \frac{du}{\sqrt{u^2+1}} = \frac{2}{3}\ln\left[\mu+\sqrt{u^2+1}\right]_0^{3^{\frac{3}{2}}}$$

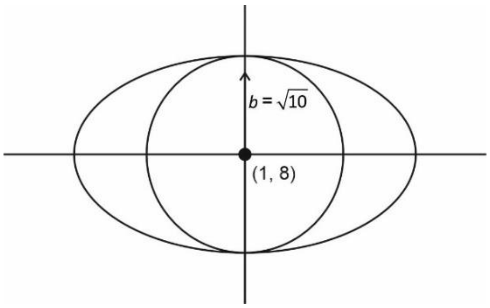
$$\boxed{\frac{2}{3}\ln(2^{3/2}+3)}$$

7. The number of values of $Z \in C$ satisfying the equations $|Z-(4+8i)| = \sqrt{10}$ and $|Z-(3+5i)| + |Z-(5+11i)| = 4\sqrt{5}$ is

(1) 0
(2) 1
(3) 2
(4) 4

**Answer (3)**

**Sol.**




$2ae = \sqrt{4+36} = \sqrt{40}$

$2a = 4\sqrt{5} \Rightarrow a = 2\sqrt{5}$

$e = \frac{\sqrt{40}}{\sqrt{80}} = \frac{1}{\sqrt{2}}$

$\frac{1}{2} = 1 - \frac{b^2}{20} \Rightarrow 10 = 20 - b^2 \Rightarrow b^2 = \sqrt{10}$

$\Rightarrow$ Circle touches ellipse at 2 points at minor axis

8. $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$

and a vector $\vec{c}$ be such that $2(\vec{a}\times\vec{b}) + 3(\vec{b}\times\vec{c}) = \vec{0}$. If $\vec{a}\cdot\vec{c} = 15$, then the value of $\vec{c}\cdot(\hat{i}+\hat{j}-3\hat{k})$ is

(1) 5
(2) –5
(3) –3
(4) 3

**Answer (2)**

**Sol.** $2(\vec{a}\times\vec{b}) + 3(\vec{b}\times\vec{c}) = \vec{0}$

$3(\vec{b}\times\vec{c}) = -2(\vec{a}\times\vec{b})$

$3(\vec{b}\times\vec{c}) = 2(\vec{b}\times\vec{a})$

$\vec{b}\times(3\vec{c}-2\vec{a}) = \vec{0}$

$\therefore \vec{b}$ & $3\vec{c}-2\vec{a}$ are collinear

$3\vec{c}-2\vec{a} = \lambda\vec{b}$

$\vec{c} = \frac{2\vec{a}+\lambda\vec{b}}{3}$

$\vec{a}\cdot\vec{c} = 15$

$\vec{a}\cdot\left(\frac{2\vec{a}+\lambda\vec{b}}{3}\right) = 15$

$2|\vec{a}|^2 + \lambda(\vec{a}\cdot\vec{b}) = 45$

$2(26) + \lambda(35) = 45$

$\lambda = -\frac{1}{5}$

$\therefore \vec{c} = \frac{2\vec{a}-\frac{1}{5}\vec{b}}{3} = \frac{10\vec{a}-\vec{b}}{15}$

$\vec{c} = \frac{30\hat{i}-12\hat{j}+31\hat{k}}{15}$

Let $\vec{d} = \hat{i}+\hat{j}-3\hat{k}$

$\vec{c}\cdot\vec{d} = \frac{1}{15}(30-12-93)$

$= -5$

9. The mean and variance of $x_1, x_2, x_3, x_4$ be 1 and 13 respectively and the mean and variance of $y_1, y_2, y_3, \ldots y_5, y_6$ be 2 and 1 respectively, then the variance of $x_1, x_2, x_3, x_4, y_1, y_2, \ldots, y_6$ will be

(1) 6.04
(2) 6.58
(3) 5.96
(4) 6.25

**Answer (1)**

**Sol.** $\dfrac{\sum_{i=1}^{4} x_i}{4} = 1 \Rightarrow \sum_{i=1}^{4} x_i = 4$ ...(1)

$$\frac{\sum_{i=1}^{4} x_i^2}{4} - 1 = 13$$

$\sum_{i=1}^{4} x_i^2 = 56$ ...(2)

$\sum_{i=1}^{6} \dfrac{y_i}{6} = 2 \Rightarrow \sum_{i=1}^{6} y_i = 12$ ...(3)

$$\frac{\sum_{i=1}^{6} \frac{y_i^2}{6}}{6} - 4 = 1$$

$\sum_{i=1}^{6} y_i^2 = 30$ ...(4)

Combine mean = $\dfrac{\sum_{i=1}^{4} x_i^2 + \sum_{i=1}^{6} y_i}{10} = \dfrac{4+12}{10} = 1.6$

Combined variance = $\dfrac{\sum_{i=1}^{4} x_i^2 + \sum_{i=1}^{6} y_i^2}{10} - (1.6)^2$

= 8.6 – 2.56 = 6.04

10. Let $O$ be the vertex of the parabola $y^2 = 4ax$, $(a > 0)$. Let $P$ and $Q$ be two variable points on the parabola such that chords $OP$ and $OQ$ are perpendicular to each other. If the locus of mid point of segment $PQ$ is a conic $C$ then length of latus rectum of $C$ is

(1) $a$

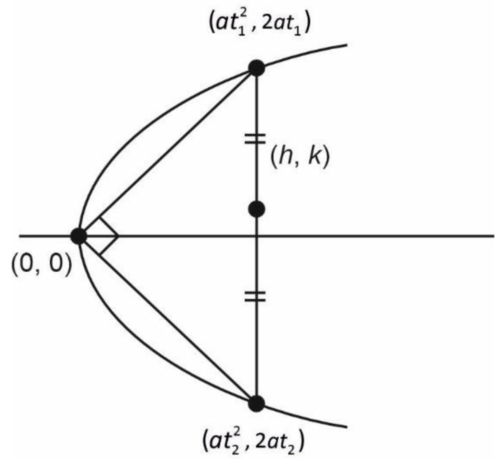
(2) $2a$

(3) $3^a$

(4) $4a$

**Answer (2)**

**Sol.**




$$\left(\frac{2at_1}{at_1^2}\right) \times \frac{2at_2}{at_2^2} = -1 \Rightarrow \boxed{t_1 t_2 = -4}$$

$$\Rightarrow 2h = a(t_1^2 + t_2^2)$$

$$2k = 2a(t_1 + t_2)$$

$$\Rightarrow t_1 t_2 = \frac{(t_1+t_2)^2 - (t_1^2+t_2^2)}{2}$$

$$= \frac{\left(\frac{k}{a}\right)^2 - \frac{2h}{a}}{2} = \frac{\frac{k^2}{a^2} - \left(\frac{2h}{a^2}\right)(a)}{2}$$

$$\Rightarrow -4 = \frac{y^2 - 2ax}{2a^2}$$

$$\Rightarrow y^2 = 2ax - 8a^2$$

$$y^2 = 2a(x - 4a)$$

$\Rightarrow$ Latus rectum = 2a

11. A student goes to examination center by bus, scooter or car probability of which being equally likely. Probability that he reaches late, if he takes bus, scooter or car is $\dfrac{1}{5}, \dfrac{1}{3}$ and $\dfrac{1}{4}$ respectively. Given that he reaches late, the probability he travelled by a bus is

(1) $\dfrac{12}{47}$

(2) $\dfrac{1}{2}$

(3) $\dfrac{26}{7}$

(4) $\dfrac{11}{6}$

**Answer (1)**

**Sol.** B: Event that he goes to exam by bus

S: Event that he goes to exam by scooter

C: Event that he goes to exam by car

L: Event that he is late

$$P(B)=P(S)=P(C)=\frac{1}{3}$$

$$P\left(\frac{L}{B}\right)=\frac{1}{5}\ P\left(\frac{L}{S}\right)=\frac{1}{3},\ P\left(\frac{L}{C}\right)=\frac{1}{4}$$

$$P\left(\frac{B}{L}\right)=\frac{P(B\cap L)}{P(L)}$$

$$=\frac{P(B)P\left(\frac{L}{B}\right)}{P(B)P\left(\frac{L}{B}\right)+P(S)P\left(\frac{L}{S}\right)+P(C)P\left(\frac{L}{C}\right)}$$

$$=\frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{4}}=\frac{\frac{1}{5}}{\frac{47}{60}}=\frac{12}{47}$$

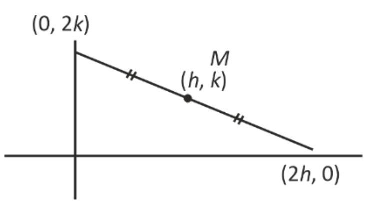
12. The line passing through points of intersection of $3x + 4y = 1$ and $4x + 3y = 1$ intersects axes at $P$ and $Q$. Then the locus of mid point of $PQ$ is

(1) $\frac{1}{x}+\frac{1}{y}=14$ (2) $\frac{3}{x}+\frac{4}{y}=14$

(3) $\frac{4}{x}+\frac{3}{y}=14$ (4) $x + y = 14$

**Answer (1)**

**Sol.**




Locus of $M\equiv\frac{x}{2h}+\frac{y}{2k}=1$

The locus passes through point of intersection of lines

$3x + 4y = 1$

$4x + 3y = 1$

$$\Rightarrow x=\frac{1}{7}, y=\frac{1}{7}$$

$$\Rightarrow \frac{1}{7}\times\frac{1}{24}+\frac{1}{7}\times\frac{1}{24}=1$$

$$\Rightarrow \boxed{\frac{1}{x}+\frac{1}{y}=14}$$

13. Consider the relation R defined on the set A = {–2, –1, 0, 1, 2} defined by (a, b) $\in$ R if and only if 1 + ab > 0. Given below are two statement:

Statement I: The number of elements in R is 17.

Statement II : R is an equivalence relation in light of the above statements, choose the correct answer.

(1) Both statement I and statement II are correct

(2) Both statement I and statement II are incorrect

(3) Statement I is correct but statement II is incorrect

(4) Statement I is incorrect but statement II is correct

**Answer (3)**

**Sol.** A = {–2, –1, 0, 1, 2}

Reflexivity $(a, a) \in$ R

as $1 + a^2 > 0$

Symmetricity $(a, b) \in$ R

$\Rightarrow 1 + ab > 0$

$\Rightarrow (b, a) \in$ R

Transitive : (–2, 0), (0, 2) $\in$ R

but (–2, 2) $\notin$ R

as 1 + (–2)(2) = –3 < 0

$\Rightarrow$ R is not an equivalence relation

For No. of elements in R

For a = −2 : (−2, −2), (−2, −1), (−2, 0) $\Rightarrow$ 3 elements

for a = −1 : (−1, −2), (−1, −1), (−1, 0) $\Rightarrow$ 3 elements

for a = 0 : (0, -2), (0, -1), (0, 0), (0, 1), (0, 2) $\Rightarrow$ 5 elements

for a = 1 : (1, 0), (1, 1), (1, 2) $\Rightarrow$ 3 elements

For a = 2 : (2, 0), (2, 1), (2, 2) $\Rightarrow$ 3 elements

Total in R elements = 17

14.

15.

16.

17.

18.

19.

20.

**SECTION - B**

**Numerical Value Type Questions:** This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the system of equation

$$x+y+z=6$$

$$x+2y+5z=10$$

$$2x+3y+\lambda z=\mu$$

has infinitely many solution then $\lambda + \mu$ is equal to

**Answer (22.00)**

**Sol.** $\Delta=\begin{vmatrix}1&1&1\\1&2&5\\2&3&\lambda\end{vmatrix}$

for infinite solution,

$\Delta=0\Rightarrow\lambda-6=0\Rightarrow\lambda=6$

also,

$$\Delta_2=\begin{vmatrix}1&1&6\\1&2&10\\2&3&\mu\end{vmatrix}$$

$\Delta_z=0\Rightarrow\mu-16=0\Rightarrow\lambda=16$

$\Rightarrow\mu+\lambda=16+6=22$

22. Let $f(x)=\begin{cases}\dfrac{1}{3}, & x<\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & x>\dfrac{\pi}{2}\end{cases}$. If $f$ is a continuous function at $x=\dfrac{\pi}{2}$, then the value of $\int_0^{3b-6}|x^2+2x-3|\,dx$ is

**Answer (4)**

**Sol.** $\lim\limits_{x\to\frac{\pi}{2}^-}f(x)=\lim\limits_{x\to\frac{\pi}{2}^+}f(x)$

LHL $=\dfrac{1}{3}$

RHL $=\lim\limits_{x\to\frac{\pi}{2}^+}\dfrac{b(1-\sin x)}{(\pi-2x)^2}$

$$=\lim_{h\to0}\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2}$$

$$=\lim_{h\to0}\frac{b(1-\cos h)}{4h^2}$$

$$=\lim_{h\to0}\frac{b\cdot2\cdot\sin^2\frac{h}{2}}{4h^2}=\frac{2b}{16}=\frac{b}{8}$$

$\therefore\ \dfrac{1}{3}=\dfrac{b}{8}\Rightarrow b=\dfrac{8}{3}$

$$I=\int_0^2|x^2+2x-3|\,dx$$

$$=\int_0^1-(x^2+2x-3)dx+\int_1^2(x^2+2x-3)dx$$

$$=\left[-\frac{x^3}{3}-\frac{2x^2}{2}+3x\right]_0^1+\left[\frac{x^3}{3}+\frac{2x^2}{2}-3x\right]_1^2$$

$$=\left(-\frac{1}{3}-1+3\right)+\left(\frac{8}{3}+4-6-\frac{1}{3}-1+3\right)$$

$$=\frac{5}{3}+\frac{7}{3}=4$$

23. If $\int_{\frac{\pi}{6}}^{\pi/4}\left(\cot\left(\pi-\frac{\pi}{3}\right)\cot\left(\pi+\frac{\pi}{3}\right)+1\right)dx$ =

$=-\alpha\left(\ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\ln\left(\frac{1}{2}\right)\right)$ then the value of $9\alpha^2$ is

**Answer (3)**

**Sol.** $\int_{\pi/6}^{\pi/4}\left(\cot\left(x-\frac{\pi}{3}\right)\cot\left(x+\frac{\pi}{3}\right)+1\right)dx$

$$\because \cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$$

$$\Rightarrow \int_{\pi/6}^{\pi/4}\cot\left(x+\frac{\pi}{3}-x+\frac{\pi}{3}\right)\left[\cot\left(x-\frac{\pi}{3}\right)-\cot\left(x+\frac{\pi}{3}\right)\right]dx$$

$$\Rightarrow \cot\frac{2\pi}{3}\int_{\pi/6}^{\pi/4}\cot\left(x-\frac{\pi}{3}\right)-\cot\left(x+\frac{\pi}{3}\right)dx$$

$$\Rightarrow -\frac{1}{\sqrt{3}}\left[\ln\left(\left|\sin\left(x-\frac{\pi}{3}\right)\right|\right)-\ln\left|\sin\left(x+\frac{\pi}{3}\right)\right|\right]_{\pi/6}^{\pi/4}$$

$$\Rightarrow -\frac{1}{\sqrt{3}}\left[\ln\left(\left|\frac{\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x+\frac{\pi}{3}\right)}\right|\right)\right]_{\pi/6}^{\pi/4}$$

$$=-\frac{1}{\sqrt{3}}\left[\ln\left(\left|\frac{\sin\left(\frac{\pi}{12}\right)}{\sin\left(\frac{7\pi}{12}\right)}\right|\right)-\ln\left(\left|\frac{\sin\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{2}\right)}\right|\right)\right]$$

$$=-\frac{1}{\sqrt{3}}\left(\ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right|-\ln\left(\frac{1}{2}\right)\right)$$

24. If the sum

$$26\left(\frac{2^3}{3}.{}^{12}C_2+\frac{2^5}{5}.{}^{12}C_4+\frac{2^7}{7}.{}^{12}C_6+\ldots\frac{2^{13}}{13}.{}^{12}C_{12}\right)$$

$=3^{13}-\alpha$ then $\alpha$ is equal to

**Answer (51.00)**

**Sol.** Notice that

$$T_r=\frac{2^{2r+1}.{}^{12}C_{2r}}{(2r+1)}$$

Notice that

$${}^{12}C_0x^0+{}^{12}C_2x^2+\ldots+{}^{12}C_{12}x^{12}$$

$$=\left(\frac{1}{2}\left[(1+x)^{12}+(1-x)^{12}\right]\right)$$

Integrating both sides

$$\int_0^2\sum_{r=0}^{6}\left({}^{12}C_{2r}x^{2r}\right)=\int_0^2\frac{1}{2}\left((1+x)^{12}+(1-x)^{12}\right)dx$$

$$\sum_{r=0}^{6}\int_0^2\frac{{}^{12}C_{2r}\cdot x^{2r+1}}{2r+1}=\frac{1}{2}\left(\frac{(1+x)^{13}}{13}\right)\Bigg|_0^2$$

$$\sum_{r=0}^{6}\frac{2^{2r+1}.{}^{12}C_{2r}}{2r+1}=\frac{1}{26}\left[3^{13}-(-1)^{13}-(1-1)\right]$$

$$\frac{2.{}^{12}C_0}{1}+\sum_{r=1}^{6}\frac{2^{2r+1}.{}^{12}C_{2r}}{2r+1}=\frac{1}{26}(3^{13}+1)$$

$$\Rightarrow S=\frac{1}{26}(3^{13}+1)-2$$

$$=\frac{3^{13}-51}{26}$$

$$\Rightarrow 26S=3^{13}-51\Rightarrow \alpha=51$$

25. Consider a circle C defined by the equation:

$x^2+y^2-6x-8y-140=0$

Let $AB$ be a variable chord of this circle such that it subtends a right angle (90°) at the origin (0, 0). If the locus of the foot of the perpendicular drawn from the origin to the chord AB is a circle given by the equation:

$x^2 + y^2 - \alpha x + \beta y - \gamma = 0$

Find the value of the expression:

$\alpha + \beta + 2\gamma$

**Answer (139)**

**Sol.** The equation of the chord AB passing through (h, k) is:

$$y - k = -\frac{h}{k}(x - h)$$

$$hx + ky = h^2 + k^2$$

$$\frac{hx + ky}{h^2 + k^2} = 1$$

The chord AB subtends a right angle at the origin (0,0). We homogenize the circle equation

$$x^2 + y^2 - 6x - 8y - 140 = 0$$

using the line equation:

$$x^2 + y^2 - (6x + 8y)\left(\frac{hx + ky}{h^2 + k^2}\right) - 140\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$$

Let $S = h^2 + k^2$. The coefficient of $x^2$ is:

$$1 - \frac{6h}{S} - \frac{140h^2}{S^2}$$

The coefficient of $y^2$ is:

$$1 - \frac{8k}{S} - \frac{140k^2}{S^2}$$

Setting their sum to zero:

$$\left(1 - \frac{6h}{S} - \frac{140h^2}{S^2}\right) + \left(1 - \frac{8k}{S} - \frac{140k^2}{S^2}\right) = 0$$

$$2 - \frac{6h + 8k}{S} - \frac{140(h^2 + k^2)}{S^2} = 0$$

Since $S = h^2 + k^2$, the third term simplifies to

$$\frac{140S}{S^2} = \frac{140}{S}:$$

Multiply the entire equation by $S$:

$$2S - (6h + 8k) - 140 = 0$$

$$2(h^2 + k^2) - 6h - 8k - 140 = 0$$

$$h^2 + k^2 - 3h - 4k - 70 = 0$$

Replacing $(h, k)$ with $(x, y)$, the locus is:

$$x^2 + y^2 - 3x - 4y - 70 = 0$$

$\Rightarrow \quad \alpha + \beta + 2\gamma = 139$

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