

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If complex numbers $Z_1, Z_2, \dots, Z_n$ satisfy the equation $4Z^2 + \bar{Z} = 0$, then $\sum_{i=1}^n |Z_i|^2$ is equal to

- (1) $\frac{3}{16}$ (2) $\frac{3}{64}$
 (3) $\frac{9}{64}$ (4) $\frac{1}{16}$

Answer (1)

Sol. $4z^2 + \bar{z} = 0$

If $z = 0$ then it is a solution

$$\Rightarrow |z| = 0$$

Now, $z \neq 0$

Multiplying by z we get

$$4z^3 + |z|^2 = 0$$

$$\Rightarrow 4z^3 = -|z|^2$$

Taking modulus both sides

$$4|z|^3 = |z|^2 \Rightarrow |z|^2(4|z| - 1) = 0$$

$$\Rightarrow |z| = \frac{1}{4}$$

$$\Rightarrow 4z^3 = \frac{-1}{16} \Rightarrow z^3 = \frac{-1}{64}$$

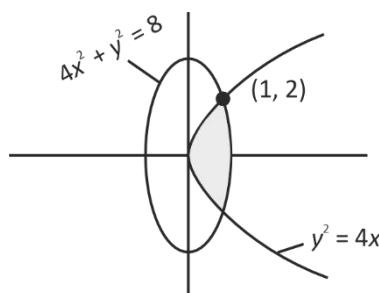
$$\Rightarrow z = \frac{-1}{4}, \frac{-1}{4}\omega, \frac{-1}{4}\omega^2$$

2. Area enclosed by $4x^2 + y^2 \leq 8$ and $y^2 \leq 4x$ is (in square unit) is

- (1) $\left(\pi + \frac{4}{3}\right)$ sq. unit (2) $\left(\pi - \frac{4}{3}\right)$ sq. unit
 (3) $\left(\pi + \frac{2}{3}\right)$ sq. unit (4) $\left(\pi - \frac{2}{3}\right)$ sq. unit

Answer (3)

Sol.



$$\begin{aligned} \text{Area} &= 2 \int_0^2 \sqrt{\frac{8-y^2}{4}} - \frac{y^2}{4} dx \\ &= \int_0^2 \left(\sqrt{8-y^2} - \frac{y^2}{2} \right) dy \\ &= 4 \sin^{-1} \left(\frac{y}{2^{3/2}} \right) + \frac{y\sqrt{8-y^2}}{2} - \frac{y^3}{6} \Big|_0^2 \\ &= 4 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) + \frac{2 \times 2}{2} - \frac{8}{6} \\ &= \frac{4\pi}{4} + 2 - \frac{4}{3} \\ &= \left(\pi + \frac{2}{3} \right) \text{ sq. unit} \end{aligned}$$

3. Mean deviation about median for $k, 2k, 3k, \dots, 1000k$ is 500, then the value of k^2 is

- (1) 4
 (2) 9
 (3) 16
 (4) 1

Answer (1)

Sol. data set

$k, 2k, 3k, \dots, 1000k$.

$$\text{Median} = \frac{500k + 501k}{2} = 500.5k$$

Mean-deviation about median

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$$500 = \frac{|500.5k - k| + |500.5k - 2k| + \dots + |1000k - 500.5k|}{1000}$$

$$= \frac{2[499.5k + 498.5k + \dots + 0.5k]}{1000}$$

$$250000 = \frac{500}{2}[0.5 + 499.5]k$$

$$\Rightarrow k = 2$$

$$k^2 = 4$$

4. If $4x^2 + y^2 < 52$, $x, y \in I$ then number of ordered pairs

(x, y) is

(1) 67

(2) 87

(3) 77

(4) 38

Answer (3)

Sol. $4x^2 + y^2 < 52$, $x, y \in I$

$$\Rightarrow 4x^2 < 52 \Rightarrow x^2 < 13 \Rightarrow x^2 \in \{0, 1, 4, 9\}$$

$$\text{If } x^2 = 0 \Rightarrow y^2 < 52 \Rightarrow y^2 \in \{0, 1, 4, 9, 16, 25, 36, 49\}$$

$$1 \times (1 + 2 \times 7) = 15$$

$$\text{If } x^2 = 1 \Rightarrow y^2 < 48 \Rightarrow y^2 \in \{0, 1, 4, 9, 16, 25, 36\}$$

$$2 \times (1 + 2 \times 6) = 26$$

$$\text{If } x^2 = 4 \Rightarrow y^2 < 36 \Rightarrow y^2 \in \{0, 1, 4, 9, 16, 25\}$$

$$2 \times (1 + 2 \times 5) = 22$$

$$\text{If } x^2 = 9 \Rightarrow y^2 < 16 \Rightarrow y^2 \in \{0, 1, 4, 9\}$$

$$2 \times (1 + 2 \times 3) = 14 \Rightarrow 77 \text{ pairs}$$

5. $x - ny + z = 6$

$$x - (n-2)y + (n+1)z = 8$$

$$(n-1)y + z = 1$$

Let n = number on the dies when rolled randomly then P (that system equation has unique solution)

= $\frac{k}{6}$, then sum of value of k and all possible value

of n is

(1) 22

(2) 21

(3) 20

(4) 24

Answer (4)

Sol. For system of equation to have unique solution

$$\Delta \neq 0$$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & 2-n & n+1 \\ 0 & n-1 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow n^2 - n - 2 \neq 0$$

$$(n-2)(n+1) \neq 0$$

$$\Rightarrow n \neq 2$$

$\therefore$ possible values of n is $\{1, 3, 4, 5, 6\}$

$$P(\text{unique solution}) = \frac{5}{6} = \frac{k}{6}$$

$$\Rightarrow k = 5$$

$\therefore$ sum of all possible value of n and k is

$$1 + 3 + 4 + 5 + 6 + 5 = 24$$

$$6. \text{ If } \lim_{x \rightarrow 0} \frac{e^{(a-1)x} + 2\cos(bx) + e^{-x}(c-1)}{x \cos x - \ln(1+x)} = 2$$

Then the value of $a^2 + b^2 + c^2$ is

(1) 8

(2) 9

(3) 10

(4) 11

Answer (3)

$$\text{Sol. } \lim_{x \rightarrow 0} \left(1 + x(a-1) + \frac{(x(a-1))^2}{2!} + \dots \right) + 2 \left(1 - \frac{b^2 x^2}{2!} + \frac{b^4 x^4}{4!} \right)$$

$$+ \frac{\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)(c-1)}{x \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right)}$$

$$\Rightarrow \frac{(1+2+(c-1)) + x(a-1-c+1) + x^2 \left(\frac{(a-1)^2}{2!} - b^2 + (c-1)\frac{1}{2} \right) + \dots}{\frac{x^2}{2} - \frac{5}{6}x^3 + \dots}$$

$$3 + c - 1 = 0$$

$$c = -2$$

$$a - c = 0$$

$$\Rightarrow a = c = -2$$

$$\frac{\frac{9}{2} - b^2 - \frac{3}{2}}{\frac{1}{2}} = 2$$

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$$\Rightarrow \frac{9}{2} - \frac{3}{2} - b^2 = 1$$

$$\Rightarrow \boxed{b^2 = 2}$$

$$a^2 + b^2 + c^2 = 4 + 4 + 2 = 10$$

7. If $P(10, 2\sqrt{15})$ lies on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and

length of latus rectum = 8, then the square of area of $\triangle PS_1S_2$ is [where S_1 & S_2 are the foci of the hyperbola]

(1) 2700

(2) 2400

(3) 1750

(4) 3600

Answer (1)

Sol. $\frac{100}{a^2} - \frac{60}{b^2} = 1$

Also $\frac{2b^2}{a} = 8$

$$\Rightarrow b^2 = 4a$$

$$\frac{100}{a^2} - \frac{60}{4a} = 1$$

$$100 - 15a = a^2$$

$$a^2 + 15a - 100 = 0$$

$$(a + 20)(a - 5) = 0$$

$$a = -20, 5$$

$$\because b^2 = 4a \Rightarrow a \text{ cannot be negative}$$

$$\Rightarrow \boxed{a = 5}$$

$$\boxed{b^2 = 20}$$

Now $H: \frac{x^2}{25} - \frac{y^2}{20} = 1$

$$e^2 = 1 + \frac{20}{25} \Rightarrow e = \frac{3}{\sqrt{5}}$$

$$F: (\pm 3\sqrt{5}, 0) \text{ i.e. } S_1 \text{ \& } S_2$$

$$\text{Area } PS_1S_2 = \frac{1}{2} \begin{vmatrix} 10 & 2\sqrt{15} \\ 3\sqrt{5} & 0 \\ -3\sqrt{5} & 0 \end{vmatrix} = \frac{1}{2} [-6\sqrt{75} - 6\sqrt{75}]$$

$$= 6\sqrt{75}$$

$$\text{Area}^2 = 2700$$

8. If a, b, c are in A.P where $a + b + c = 1$ and $a, 2b, c$ are in G.P., then the value of $9(a^2 + b^2 + c^2)$ is equal to

(1) 3

(2) -3

(3) 4

(4) -4

Answer (2)

Sol. $a, b, c \rightarrow AP$

$$a, 2b, c \rightarrow GP \Rightarrow 4b^2 = ac$$

$$a + b + c = 1$$

$$2b = a + c$$

$$\Rightarrow b = \frac{1}{3}$$

$$a + c = \frac{2}{3}$$

$$4b^2 = ac$$

$$\frac{4}{9} = ac$$

$$a^2 + b^2 + c^2$$

$$\Rightarrow \frac{1}{9} + (a + c)^2 - 2ac$$

$$\Rightarrow \frac{1}{9} + \frac{4}{9} - 2\left(\frac{4}{9}\right) = \frac{-1}{3}$$

$$\text{Here } a = \frac{1}{3} + \frac{i}{\sqrt{3}} \quad b = \frac{1}{3} \quad c = \frac{1}{3} - \frac{i}{\sqrt{3}}$$

$$\therefore a^2 + b^2 + c^2 \text{ is negative}$$

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9. Let the domain of function

$$f(x) = \log_3 \log_5 (7 - \log_2 (x^2 - 10x + 15)) + \sin^{-1} \left(\frac{3x-7}{17-x} \right)$$

be $(\alpha, \beta]$, then $\alpha + \beta$ is equal to

- (1) 6 (2) 7
(3) 8 (4) 9

Answer (4)

Sol. $\log_5 (7 - \log_2 (x^2 - 10x + 85)) > 0$

$$\Rightarrow 7 - \log_2 (x^2 - 10x + 85) > 1$$

$$\Rightarrow \log_2 (x^2 - 10x + 85) < 6$$

$$\Rightarrow x^2 - 10x + 85 < 64$$

$$\Rightarrow x^2 - 10x + 21 < 0$$

$$(x-7)(x-3) < 0$$



$$\Rightarrow x \in (3, 7)$$

$$\left| \frac{3x-7}{17-x} \right| \leq 1$$

$$\Rightarrow |3x-7| \leq |17-x|$$

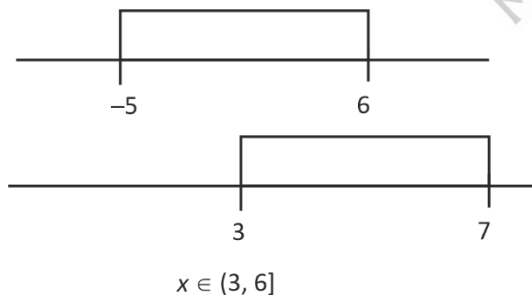
$$\Rightarrow 9x^2 + 49 - 42x \leq 289 + x^2 - 34x$$

$$\Rightarrow 8x^2 - 8x - 240 \leq 0$$

$$\Rightarrow x^2 - x - 30 \leq 0$$

$$(x-6)(x+5) \leq 0$$

$$x \in [-5, 6]$$



$$x \in (3, 6]$$

10. Let $\vec{a} = \sqrt{2}\hat{i}$ and $\vec{b} = 5\hat{j} + \hat{k}$. If $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ which lie on y - z plane such that $|\vec{d}| = 2$. Then the maximum value of $|\vec{c} \cdot \vec{d}|^2$ is equal to

- (1) 104 (2) 52
(3) 208 (4) 120

Answer (3)

Sol. $|\vec{a} \times \vec{b}| = \sqrt{52} = |\vec{c}|$

The maximum value of $|\vec{c} \cdot \vec{d}|^2$

$$= (\sqrt{52} \cdot (2) \cos \theta)^2 = 208 \cos^2 \theta$$

The maximum value of $|\vec{c} \cdot \vec{d}|^2 = 208$

11. If $\int_0^{64} \left(x^{\frac{1}{3}} + \text{floor}(x^{1/3}) \right) dx = \alpha$ and

$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin^6 x + \cos^6 x} dx = \pi\beta$, then the value of $\alpha\beta^2$ is equal to

- (1) 87 (2) 77
(3) 67 (4) 57

Answer (1)

Sol. $I_1 = \int_0^{64} \left(x^{\frac{1}{3}} + \text{floor}(x^{1/3}) \right) dx$

$$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} \Big|_0^{64} + \int_0^1 0 dx + \int_1^8 (1) dx + \int_8^{27} 2 dx + \int_{27}^{64} (3) dx$$

$$= 192 + 0 + (8-1) + 2 + 3(64-27) = 348$$

$$I_2 = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin^6 x + \cos^6 x} dx$$

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$$\Rightarrow I_2 = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin^6 x + \cos^6 x} dx$$

$$\Rightarrow 2I_2 = \int_0^{\frac{\pi}{2}} \frac{1}{\sin^6 x + \cos^6 x} dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1 + \tan^6 x} dx$$

$$\Rightarrow I_2 = \frac{\pi}{2} \Rightarrow \beta = \frac{1}{2}, \alpha = 348$$

$$\alpha\beta^2 = 348 \times \frac{1}{4} = 87$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If α, β are roots of equation $12x^2 - 20x + 3\lambda = 0$,

$\lambda \in I$. If $\frac{1}{2} \leq |\beta - \alpha| \leq \frac{3}{2}$, then the sum of all possible values of λ is

Answer (03.00)

Sol. $|\beta - \alpha|^2 \in \left[\frac{1}{4}, \frac{9}{4}\right]$

$$(\alpha^2 + \beta^2 - 2\alpha\beta) = (\alpha + \beta)^2 - 4\alpha\beta \in \left[\frac{1}{4}, \frac{9}{4}\right]$$

$$= \left(\frac{20}{12}\right)^2 - 4\left(\frac{3\lambda}{12}\right) \in \left[\frac{1}{4}, \frac{9}{4}\right]$$

$$\Rightarrow \frac{25}{9} - \lambda \in \left[\frac{1}{4}, \frac{9}{4}\right]$$

$$\Rightarrow 100 - 36\lambda \in [9, 81]$$

$$36\lambda - 100 \in [-81, -9]$$

$$36\lambda \in [19, 91]$$

$$\lambda \in \left[\frac{19}{36}, \frac{91}{36}\right]$$

then integral values of λ is 1, 2

$\Rightarrow$ Sum of integral values of λ is $1 + 2 = 3$

22. If $\cos(\alpha + \beta) = -\frac{1}{10}$ and $\sin(\alpha - \beta) = \frac{3}{8}$, where

$$0 < \alpha < \frac{\pi}{3} \text{ and } 0 < \beta < \frac{\pi}{4}, \text{ if } \tan 2\alpha = \frac{3(1 - r\sqrt{55})}{\sqrt{11}(s + \sqrt{5})}$$

and $r, s \in N$, then $r^2 + s$ is

Answer (20)

Sol. $0 < \alpha < \beta < \frac{7\pi}{12}$

$$\therefore \cos(\alpha + \beta) = -\frac{1}{10} \Rightarrow \alpha + \beta \in \text{II}^{\text{nd}} \text{ quadrant}$$

$$\sin(\alpha + \beta) = \frac{\sqrt{99}}{10}$$

$$\sin(\alpha - \beta) = \frac{3}{8} \quad \alpha + \beta \in \text{I quadrant}$$

$$\cos(\alpha - \beta) = \frac{\sqrt{55}}{8}$$

$$\tan((\alpha + \beta) + (\alpha - \beta)) = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)}$$

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$$\tan 2\alpha = \frac{-\sqrt{99} + \frac{3}{\sqrt{55}}}{1 + 3\sqrt{\frac{99}{55}}} = \left[\frac{-3\sqrt{11.55} + 3}{\sqrt{55} + 9\sqrt{11}} \right]$$

$$= \frac{3 - 3\sqrt{11} \cdot \sqrt{55}}{\sqrt{55} + 9\sqrt{11}} = \frac{3(1 - \sqrt{11}\sqrt{55})}{\sqrt{11}(\sqrt{5} + 9)}$$

$$r^2 = 11$$

$$s = 9$$

$$r^2 + s = 20$$

23.

24.

25.



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