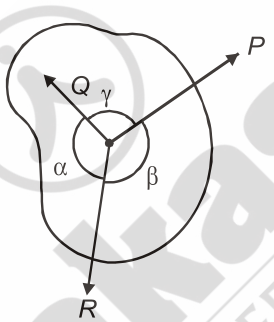
Chapter 3

Laws of Motion

Equilibrium of Concurrent Forces

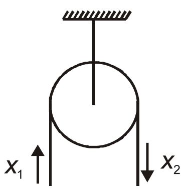
If three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are acting on an object such that forces are concurrent and the object is in equilibrium then $\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{R}{\sin\gamma}$.



APPLICATIONS OF NEWTON'S LAWS OF MOTION

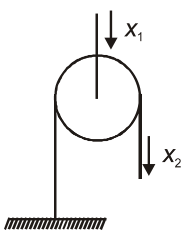
The strings connected to pulley are considered as ideal. Their length is fixed, so the ends of string follow a fixed relation between displacement velocity and acceleration. These relations are called constraint relation.

***Case I*:** When the middle end is fixed

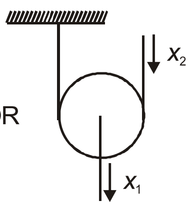


$x_1 = x_2$
$v_1 = v_2$
$a_1 = a_2$

***Case II*:** When the side end is fixed

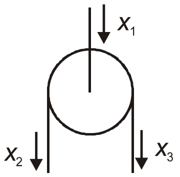


OR



$x_2 = 2x_1$
$v_2 = 2v_1$
$a_2 = 2a_1$

***Case III* :** When all the three ends are free to move



$$x_1 = \frac{x_2 + x_3}{2}$$

$$v_1 = \frac{v_2 + v_3}{2}$$

$$a_1 = \frac{a_2 + a_3}{2}$$

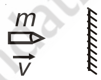
> ***Note :*** *In all the above relations downward direction is taken as positive. If any of the direction is upward in any case then –ve sign must be incorporated in the corresponding equation.*

1. A machine gun fires n bullet per second with speed u and mass of each bullet is m.

The Force required to keep the gun stationary is

$\vec{F} = nm\vec{v}$

2. Bullets moving with a speed v hit a wall normally.



(i) If the bullets come to rest in wall

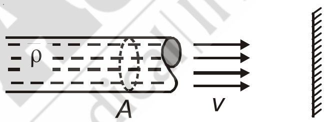
Force on wall $F_{wall} = nmv$

(Here n is number of bullets hitting the wall in one second)

(ii) If the bullets rebound elastically,

$F_{wall} = 2nmv$

3. Liquid jet of area A moving with speed v hits a wall



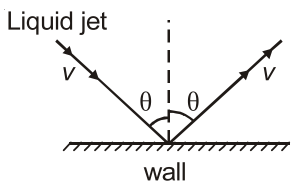
(i) Force required by a pump to move the liquid with this speed is

$$F = v\frac{dm}{dt} = v \times \rho A v = \rho A v^2$$

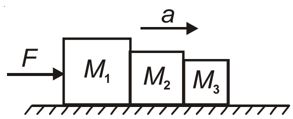
(ii) As jet hits a vertical wall and does not rebound, the force exerted by it on the wall is, $F_{wall} = \rho A v^2$

(iii) When water rebounds elastically, $F_{wall} = 2\rho A v^2$

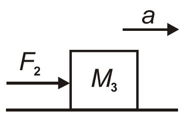
(iv) For oblique impact as shown, $F_{wall} = 2\rho A v^2 \cos\theta$



4. The blocks shown are being pushed by a force F. F_1, F_2 are contact forces between M_1 & M_2 and M_2 & M_3 respectively

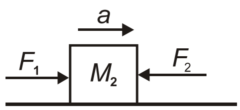


$$a = \frac{F}{M_1 + M_2 + M_3}$$

FBD of M_3 

$F_2 = M_3a$

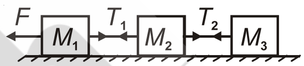
$F_1 - F_2 = M_2a \Rightarrow F_1 = (M_3 + M_2)a$

FBD of M_2 

or $F_2 = \dfrac{M_3}{M_1 + M_2 + M_3}F$, $F_1 = \left(\dfrac{M_2 + M_3}{M_1 + M_2 + M_3}\right)F$

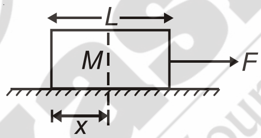
5. The strings are massless. Let T_1 and T_2 be the tensions in the two strings and 'a' be the acceleration.

$$a = \frac{F}{M_1 + M_2 + M_3},\ T_1 = \frac{(M_2 + M_3)F}{M_1 + M_2 + M_3},\ T_2 = \frac{M_3F}{M_1 + M_2 + M_3}$$

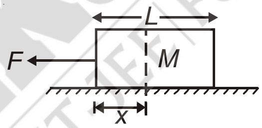


6. Tension in the block at a distance x from left end is given as

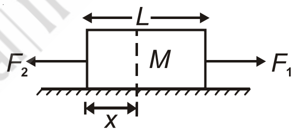
(a) $T_x = \dfrac{Mx}{L} \times \dfrac{F}{M} = \dfrac{Fx}{L}$

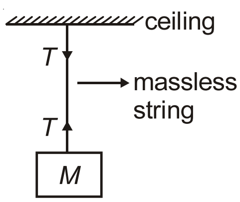


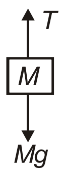
(b) $T_x = \dfrac{F(L - x)}{L}$



(c) $T_x = \dfrac{F_1x}{L} + \dfrac{F_2(L - x)}{L}$



7. 



FBD of M w.r.t. ground

(a) When system is stationary

$T - Mg = 0$

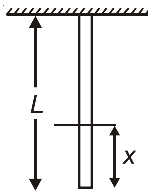
(b) System moves up with acceleration 'a'

$T = M(g + a)$

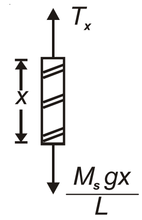
(c) System moves down with acceleration a

$T = M(g - a)$

8. Uniform rope of mass M_s.



FBD of lower portion



The tension in the rope at a distance x from free end is given below for different cases.

(a) Stationary system

$$T_x = \frac{M_s gx}{L}$$

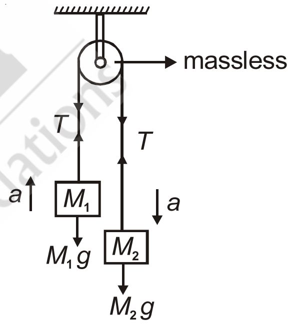
(b) If the rope is accelerating upwards, then $T_x = \frac{M_s x}{L}(g + a)$

(c) If the rope is accelerating downwards, then $T_x = \frac{M_s x}{L}(g - a)\ (g > a)$

IMPORTANT PROBLEMS

Pulley mass systems

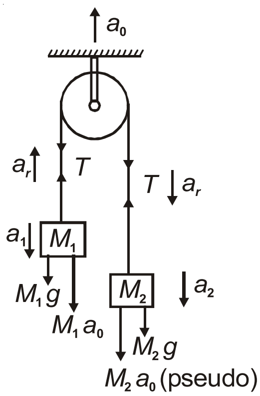
(i) Stationary pulley



$$a = \frac{M_2 - M_1}{M_2 + M_1} g$$

$$T = 2\left(\frac{M_1 M_2}{M_1 + M_2}\right) g$$

(ii) Pulley is moving upward with acceleration a_0



$$T = 2\left(\frac{M_1 M_2}{M_1 + M_2}\right)(g + a_0)$$

The acceleration of each block with respect to pulley is

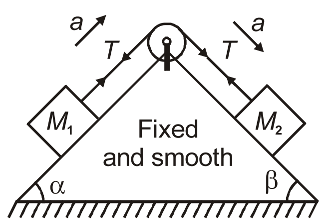
$$a_r = \left(\frac{M_2 - M_1}{M_2 + M_1}\right)(g + a_0)$$

The absolute accelerations of the two blocks are a_1 and a_2

$$a_1 = -(a_0 + a_r) \qquad \left\{ a_0 = \left(\frac{a_1 + a_2}{2}\right) \right\}$$

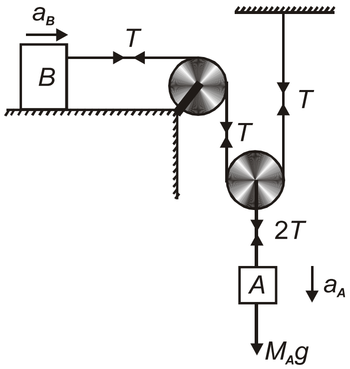
$$a_2 = a_r - a_0$$

(iii) $T = \frac{M_1 M_2 g}{M_1 + M_2}(\sin\alpha + \sin\beta)$



$$a = \left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_2 + M_1}\right) g$$

(iv)



$2a_A = a_B$ ...(i)

$M_Ag - 2T = M_Aa_A$ ...(ii)

$T = M_Ba_B$ ...(iii)

Two block system :

Case - I :

Let '*m*' does not slide down relative to wedge '*M*'

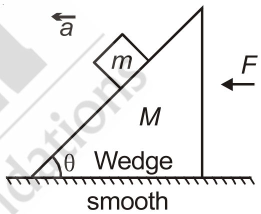
The force required is given by

$F = (M + m)g \tan\theta$

$a = g \tan \theta$ (in horizontal direction w.r.t. ground)

Contact force *R* between *m* and *M* is

$$R = \frac{mg}{\cos\theta}$$



Case - II :

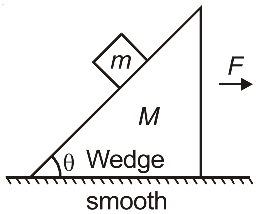
Minimum value of *F* so that '*m*' falls freely is given by

$F = Mg\cot\theta$

Wedge *M* moves with acceleration = $g\cot\theta$

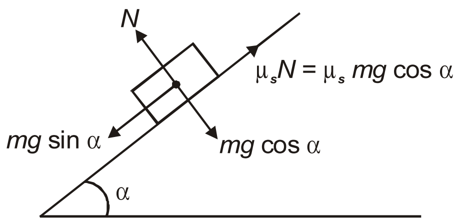
The block falls vertically with acceleration '*g*'.

Contact force between *M* and *m* is zero.



Angle of Repose

Consider a situation in which a block is placed on an inclined plane with co-efficient of friction 'μ'. The maximum value of angle of inclined plane for which the block can remain at rest is defined as angle of repose.

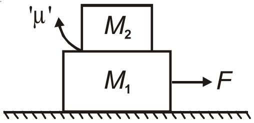


$\alpha = \tan^{-1}(\mu_s)$

Two blocks placed one above the other on smooth ground

Case - I : If a force F is applied on lower block

(a) $F \le \mu(M_1 + M_2)g$



Smooth

Both blocks move together with same acceleration $a = \dfrac{F}{M_1 + M_2}$

$$a_{max} = \mu g$$

(b) $F > \mu(M_1 + M_2)g$

M_2 moves with constant acceleration $a_2 = \mu g$

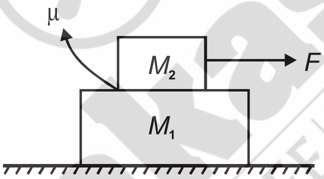
M_1 moves with acceleration $a_1 = \dfrac{F - \mu M_2 g}{M_1}$

M_2 slips backward on M_1.

Case - II : If a force F is applied on upper block

(a) $F \le \dfrac{\mu(M_1 + M_2)M_2}{M_1} g$, both blocks move together with acceleration $a = \dfrac{F}{M_1 + M_2}$ with $a_{max} = \dfrac{\mu M_2}{M_1} g$.

(b) If $F > \dfrac{\mu(M_1 + M_2)M_2 g}{M_1}$,



Smooth

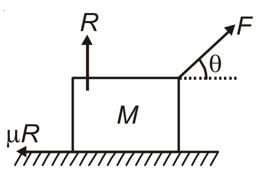
M_1 moves with constant acceleration $a_1 = \dfrac{\mu M_2}{M_1} g$

M_2 moves with acceleration $a_2 = \dfrac{F - \mu M_2 g}{M_2}$

M_2 slips forward on M_1.

Minimum force required to move a body on a rough horizontal surface

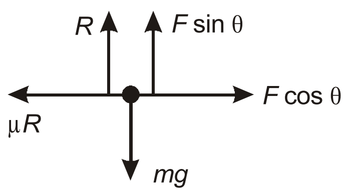
$F \cos\theta > \mu R \quad (R = mg - F\sin\theta)$



$$F \ge \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

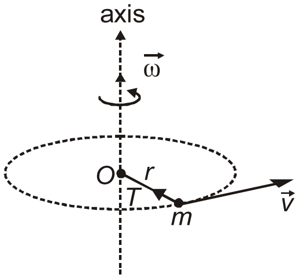
$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} \text{ at } \theta = \tan^{-1}(\mu)$$

$\Rightarrow$



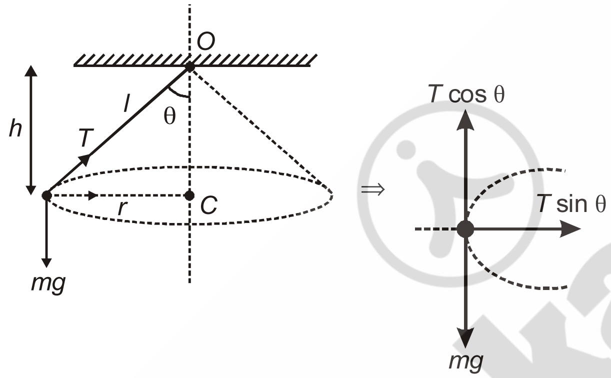
DYNAMICS OF CIRCULAR MOTION

Neglecting Gravity



T = Centripetal force = $\dfrac{mv^2}{r} = m\omega^2 r$

Considering gravity (Conical pendulum)



$T\sin\theta = m\omega^2 r$ ...(1)

$T\cos\theta = mg$

$T = \dfrac{mg}{\cos\theta}$ ...(2)

(a) For θ to be 90º (*i.e.*, string to be horizontal)

$T = \infty$

∴ It is not possible.

(b) $T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$

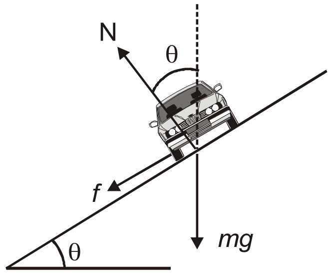
⇒ $T = m\omega^2 l$

(c) Time period = $2\pi\sqrt{\dfrac{h}{g}}$ ($h = l\cos\theta$)

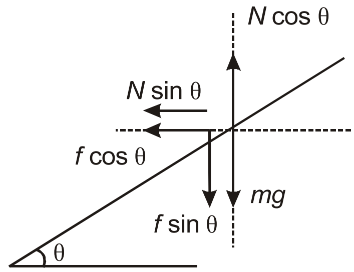
Vehicle negotiating a curve on a banked road

The maximum velocity with which a vehicle can safely negotiate a curve of radius r on a rough inclined road is

$$V_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$



$\Rightarrow$



Special Cases :

For a smooth inclined surface $\mu = 0 \Rightarrow$ $v_{max} = \sqrt{rg\tan\theta}$

For a horizontal rough surface, $\theta = 0 \Rightarrow$ $v_{max} = \sqrt{\mu rg}$

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