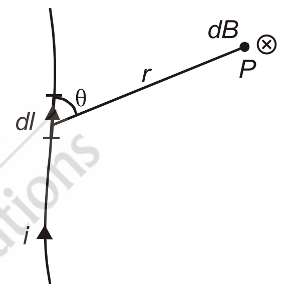
Chapter 12

Magnetic Effects of Current and Magnetism

MAGNETIC EFFECTS OF CURRENT

BIOT-SAVART LAW



Magnetic field due to current carrying element is given by

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i(\vec{dl}\times\vec{r})}{r^3}$$

The direction of magnetic field due to small element *dl* is in the direction of $\vec{dl}\times\vec{r}$

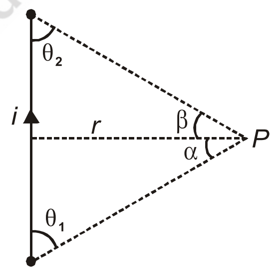
Units : 1. S.I. unit of magnetic field is tesla (T)

2. CGS unit of magnetic field is gauss.

3. 1 gauss = 10^{-4} tesla.

Magnetic Field Due to Straight Current Carrying Wire

Magnetic field at *P*



$$B = \frac{\mu_0 i}{4\pi r}(\cos\theta_1 + \cos\theta_2)$$

or $$B = \frac{\mu_0 i}{4\pi r}(\sin\alpha + \sin\beta)$$

1. For an infinite long wire

$\theta_1 = \theta_2 = 0$ or $\alpha = \beta = \frac{\pi}{2}$, $B = \frac{\mu_0 i}{2\pi r}$

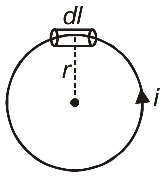
2. For semi-infinite long wire

$\theta_1 = 0°$, $\theta_2 = 90°$, or $\alpha = \frac{\pi}{2}, \beta = 0$, $B = \frac{\mu_0 i}{4\pi r}$

CIRCULAR LOOP

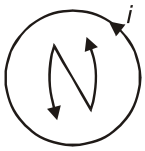
1. At the centre of a current loop

$$dB = \frac{\mu_0 i dl \sin 90^\circ}{4\pi r^2}$$

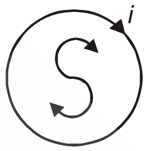


$$B = \frac{\mu_0 i}{4\pi r^2}\int dl = \frac{\mu_0 i}{4\pi r^2}\times 2\pi r$$

$$B = \frac{\mu_0 i}{2r} \text{ (Outward).}$$

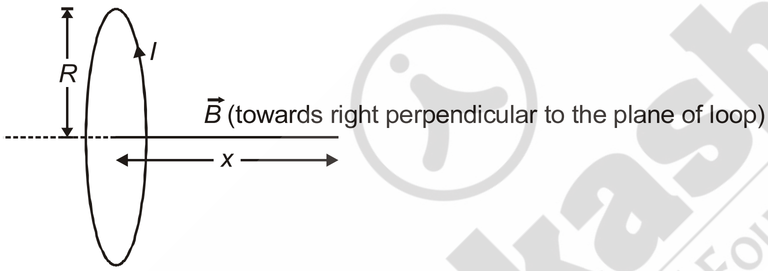


(Outward field perpendicular to the plane of loop)



(Inward field perpendicular to the plane of loop)

2. On the axis of a loop



$$B = \frac{\mu_0}{4\pi}\frac{2\times I\times \pi R^2}{(R^2+x^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2M}{(R^2+x^2)^{3/2}}$$

For $x >> R$, $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$ [Current carrying loop acts as an magnetic dipole]

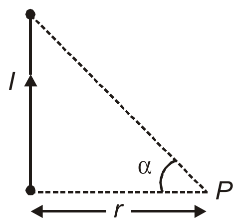
where $M = I \times \pi R^2$ is called magnetic dipole moment

$\vec{M} = I\vec{A}$

S.I. Unit : A-m^2

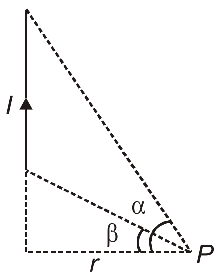
Various Cases of Magnetic Field (Straight Wire and Circular Loop)

1.



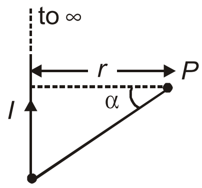
$$B_P = \frac{\mu_0 I}{4\pi r}[\sin\alpha] \otimes$$

2.



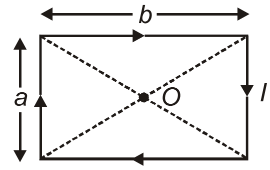
$$B_P = \frac{\mu_0 I}{4\pi r}[\sin\alpha - \sin\beta] \otimes$$

3.



$$B_P = \frac{\mu_0 I}{4\pi r}[\sin\alpha + 1] \otimes$$

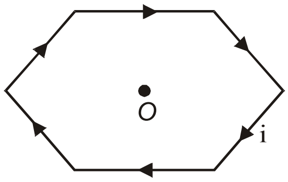
4.



$$B_0 = \frac{8\mu_0 I}{4\pi}\cdot\frac{\sqrt{a^2+b^2}}{ab}\otimes$$

when $a = b$ $B_0 = \dfrac{8\sqrt{2}\mu_0 I}{4\pi a}\otimes$

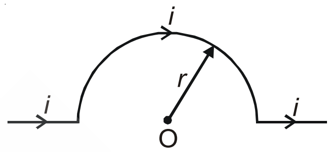
5.



Magnetic field at O

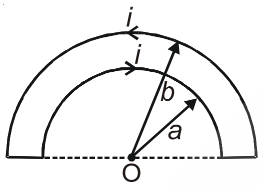
$B_0 = \dfrac{\sqrt{3}\mu_0 i}{\pi a}\otimes$ where 'a' is length of each side of regular hexagon

6.



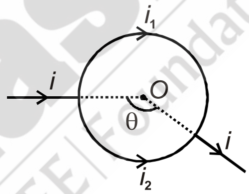
$$B_0 = \frac{\mu_0 i}{4r}\otimes$$

7.



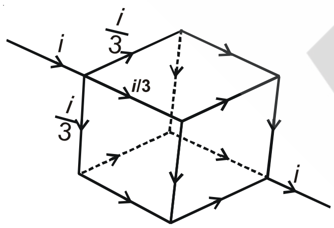
$$B_0 = \frac{\mu_0 i}{4}\left(\frac{1}{a} - \frac{1}{b}\right)\otimes$$

8.



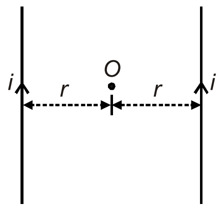
$B_0 = 0$ (for any value of θ)

9.



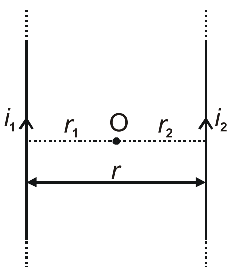
At the centre of cube, $B = 0$

10.



At O, $B = 0$

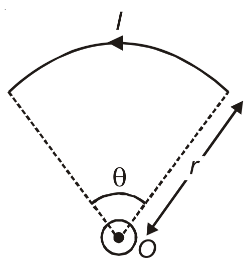
11.



At O, $B = 0$, such that

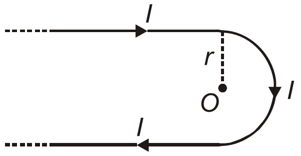
$$r_1 = \frac{i_1}{i_1 + i_2}r; r_2 = \frac{i_2 r}{i_1 + i_2}$$

12.

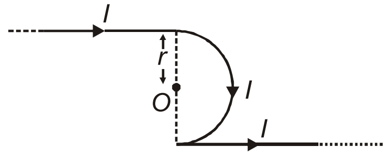


$$B_0 = \frac{\mu_0 I}{2r}\times\frac{\theta}{2\pi}\odot$$

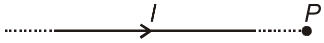
(θ is radian)

13. 

$$B_0 = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{4r} \otimes$$

14. 

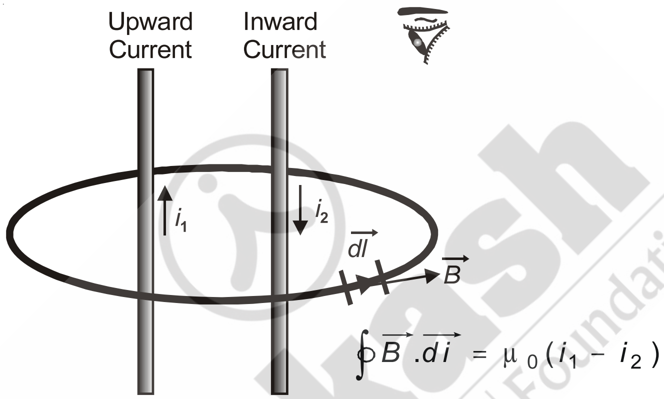
$$B_0 = \frac{\mu_0 I}{4r} \otimes$$

15. 

$B_P = 0$ (at point P)

AMPERE CIRCUITAL LAW

It states that the line integral of resultant magnetic field over a closed path is equal to μ_0 times the algebraic sum of the current threading the closed path in free space. Ampere's circuital law has the form

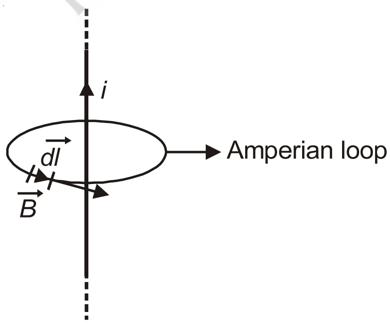


$$\oint \vec{B}.\vec{dl} = \mu_0 i_{enc}$$

Here $\oint \vec{B}.\vec{dl}$ implies the integration of scalar product $\vec{B}.\vec{dl}$ around a closed loop called an **Amperian loop**. The current i_{enc} is the net current encircled by the loop.

Applications of Ampere's Circuital Law

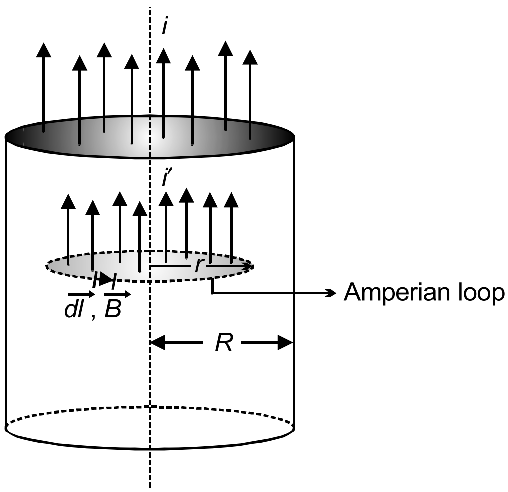
(1) Magnetic field due to a long thin current carrying wire

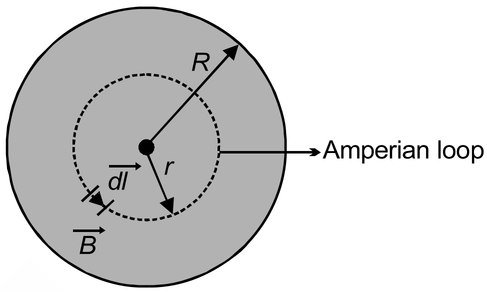


$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(2) Magnetic field inside a long straight current carrying conductor



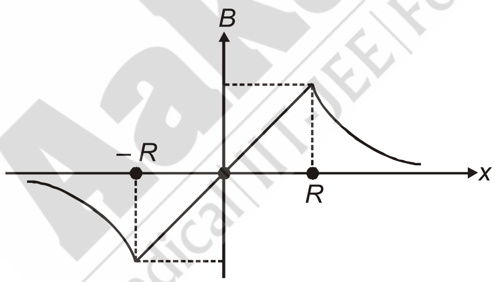


$$I' = \frac{I}{\pi R^2} \times \pi r^2$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(\frac{I}{\pi R^2} \pi r^2 \right)$$

$$B = \frac{\mu_0}{2\pi} \frac{r}{R^2} \qquad i.e.\ B_{in} \propto r$$

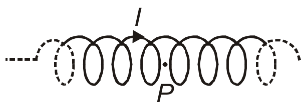
Graphical variation of magnetic field



(3) Inside a hollow tube of current, magnetic field is zero.

SOLENOID

A long solenoid having number of turns/length 'n' carries a current I.

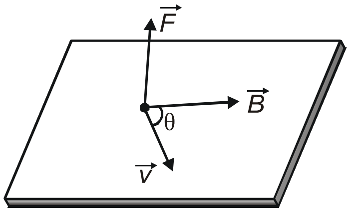


The magnetic field B is given by,

$B_P = \mu_0 nI$ (in between)

$$B_{end} = \frac{\mu_0 nI}{2} \text{ (near one end)}$$

Force on a Moving Charge in Magnetic Field



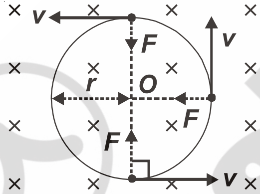
$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB \sin\theta$

$\vec{F}$ is perpendicular to both $\vec{v}$ & $\vec{B}$ and is in the direction of $\vec{v} \times \vec{B}$ when $q > 0$ and opposite to $\vec{v} \times \vec{B}$ when $q < 0$.

MOTION OF CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD $\vec{B}$.

Case-1 : $\vec{v} \perp \vec{B}$

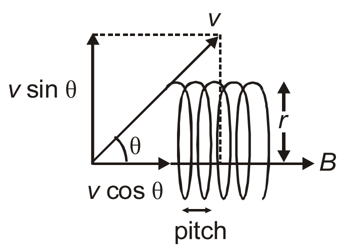


$$F = qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2km}}{qB}$$ (k is kinetic energy). As $k = \frac{p^2}{2m}$.

Results

1. During revolution its speed is constant
2. During revolution its kinetic energy is constant
3. Work done by the magnetic force is zero
4. Velocity and momentum change continuously in direction, not in magnitude.
5. $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ (Independent of speed and radius)
6. Frequency $f = \frac{qB}{2\pi m}$ (Cyclotron frequency)

Case-2 : If $\vec{v}$ is not perpendicular to $\vec{B}$. Let θ be the angle between $\vec{v}$ and $\vec{B}$.



The particle moves in a helical path such that

$$r = \frac{mv\sin\theta}{qB}, T = \frac{2\pi m}{qB}, \text{ Pitch} = v\cos\theta \times T$$

Case-3 : If charged particle is moving parallel or anti-parallel to field $\vec{B}$ then force is zero and it moves in a straight line

Cyclotron

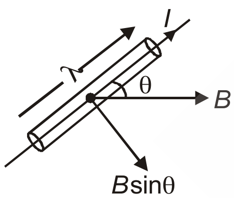
A cyclotron is a machine used to accelerate charge particle (like proton, deutron, α-particle). It uses both electric and magnetic field. Speed of particle is increased by electric field.

$$\text{Cyclotron frequency} = \frac{qB}{2\pi m}$$

$$K_{\max} = \frac{q^2B^2R_{\max}^2}{2m}$$

To accelerate electrons Betatron and Synchrotron are used. A synchrotron accounts for the variation in mass with speed. A Betatron uses the induced electric field produced by a time varying magnetic field to accelerate charged particles.

Force on a current carrying conductor in uniform field



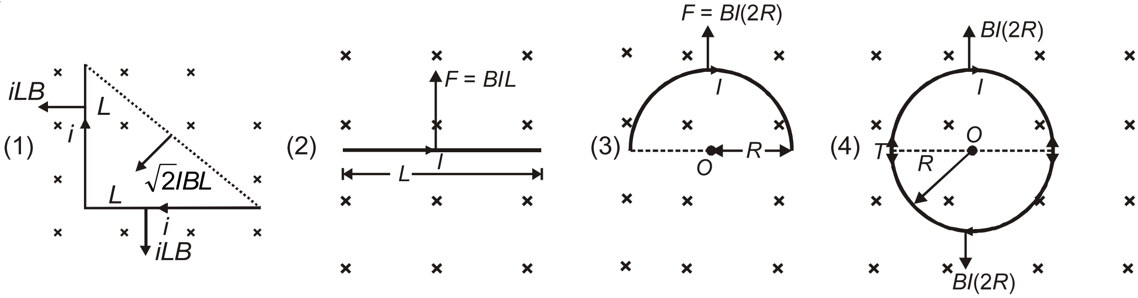
I = Current through the conductor, ℓ = Length of the conductor

$$F = IB\ell\sin\theta$$

$$\vec{F} = I(\vec{\ell} \times \vec{B})$$

The direction of $\vec{\ell}$ is always in the direction of current.

Some Important Cases

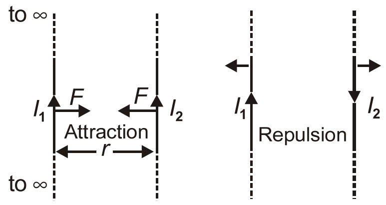


Net $F = 0$

$T = BIR$

(tension in the wire)

FORCE BETWEEN TWO CURRENT CARRYING WIRES



F (force/unit length) is given by $F = \dfrac{\mu_0 I_1 I_2}{2\pi r}$.

The force on a segment of length 'l' is $= \dfrac{\mu_0 I_1 I_2}{2\pi r} \times l$

Force on a small current carrying segment placed near long and perpendicular current carrying wire.

$$F_{PQ} = \frac{\mu_0 i_1 i_2}{2\pi} \log\left(1 + \frac{l}{d}\right)$$

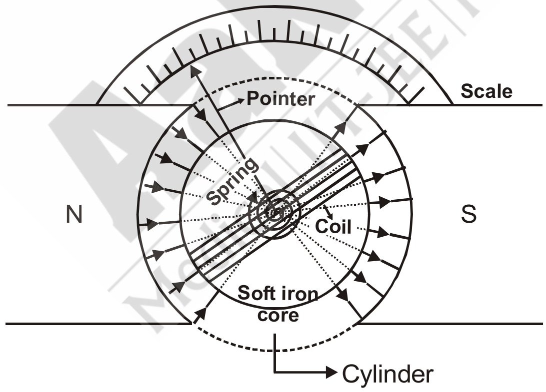
$$\tau_{PQ} \neq 0$$

MOVING COIL GALVANOMETER

It is a device used to measure small current through the circuit.

Principle

When a current carrying coil is suitably placed in a magnetic field, torque acts on it. In moving coil galvanometer radial field is used which is obtained from magnet having concave shape poles. In this type of field plane of the coil is always parallel to the magnetic field so maximum torque acts on it.



$\tau = NIAB$ [$\phi = 90°$ due to radial field] (ϕ – Angle between $\vec{M}$ and $\vec{B}$)

$= C\theta$ (θ = Angle of twist)

where θ is angle turned by the pointer and C is restoring torque/twist in the suspension wire.

$$I = \frac{C\theta}{NBA}$$

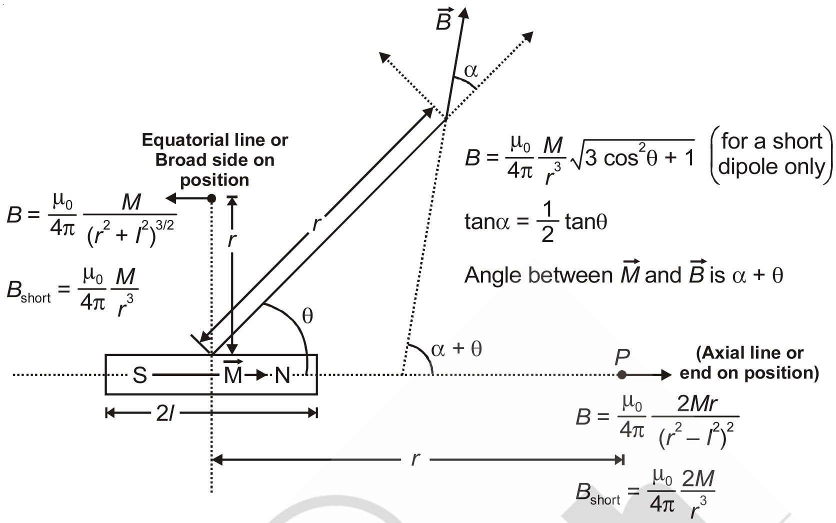
Current sensitivity = $\dfrac{\theta}{I} = \dfrac{NBA}{C}$

Voltage sensitivity = $\dfrac{\theta}{V} = \dfrac{\theta}{IR} = \dfrac{NBA}{CR}$

MAGNETISM

BAR MAGNET

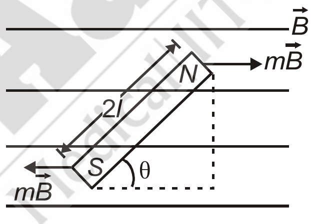
Magnetic field due to a Bar Magnet



1. $\vec{B}_{axial}$ is parallel to $\vec{M}$.
2. $\vec{B}_{equatorial}$ is antiparallel to $\vec{M}$.
3. $\vec{B} \perp \vec{M}$ when $\theta + \alpha = 90°$ *i.e.*, $\theta = 90° - \alpha$

   as $\tan\alpha = \frac{1}{2}\tan\theta \Rightarrow \cot\theta = \frac{1}{2}\tan\theta$ or $\tan\theta = \sqrt{2}$.

TORQUE ON A BAR MAGNET IN MAGNETIC FIELD



$\tau = mB \times 2l \sin\theta$

$= m \times 2l \times B \sin\theta$

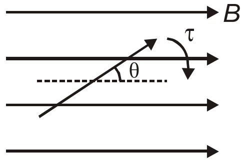
$\Rightarrow \tau = MB \sin\theta$

Results :

1. $\vec{\tau} = \vec{M} \times \vec{B}$, $\tau_{max} = MB$ [when $\theta = 90°$],
   $\tau_{min} = 0$ [when $\theta = 0$ or $180°$]
2. Net force on dipole is zero.
3. $U = -\vec{M}.\vec{B}$ $U_{min} = -MB$ at $\theta = 0°$
   $U_{max} = MB$ at $\theta = 180°$
4. Work done by external agent in rotating bar magnet from angular position θ_1 to θ_2 is
   $W = MB\,[\cos\theta_1 - \cos\theta_2]$
5. A bar magnet kept in a non-uniform magnetic field experience a net force and may experience a torque.

Oscillations of a Bar Magnet in Magnetic Field

For small displacements from equilibrium position, bar magnet oscillates simple harmonically such that



Angular frequency $\omega = \sqrt{\frac{MB}{I}}$;

Time period $T = 2\pi\sqrt{\frac{I}{MB}}$

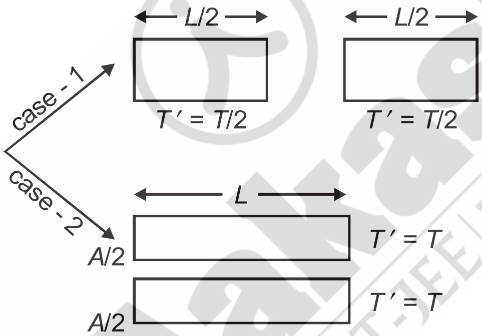
Some important cases related to time period are given below

A bar magnet of length L is,

Pole strength ∝ Area of cross-section of magnetic dipole ($m \propto A$)

$\vec{M} = m \times 2\vec{l}$

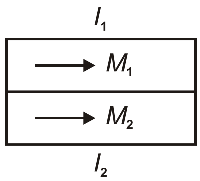
1. Cut into two halves



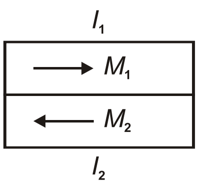
where $T = 2\pi\sqrt{\frac{I}{MB}}$

2. Two bar magnets having magnetic moments M_1, M_2 and moment of inertias I_1, I_2 are joined as shown.

(a) $T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}}$



(b) $T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}}$

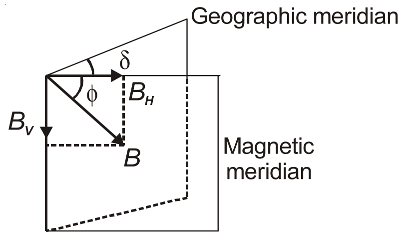


$\Rightarrow \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{M_1}{M_2}$

EARTH'S MAGNETIC FIELD

The basic components of earth's magnetic field at a place are shown

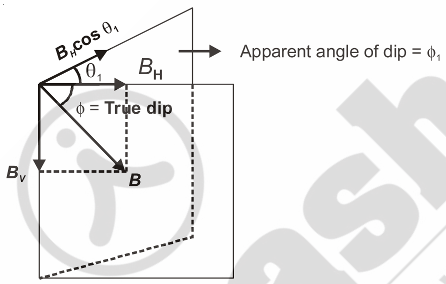
1. δ = Angle of declination
2. ϕ = Angle of dip/inclination
3. $B_H = B \cos \phi$
4. $B_V = B \sin \phi$
5. $B_H^2 + B_V^2 = B^2$



Note : *The needle of a vertical compass in magnetic meridian points toward B.*

6. $\tan\phi = \dfrac{B_V}{B_H}$

7. When the dip circle is not in magnetic meridian and dip circle is at an angle of θ_1 to magnetic meridian.



[ϕ = True dip angle, ϕ_1 and ϕ_2 = apparent dip angle in two arbitrary perpendicular)

$\tan\phi_1 = \dfrac{B_V}{B_H \cos\theta_1}$ (apparent dip) [Vertical component remains same]

$\Rightarrow \quad \tan\phi_1 = \dfrac{\tan\phi}{\cos\theta_1}$

8. $\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$, plane

PARA, DIA AND FERROMAGNETIC SUBSTANCES

All the elements of the nature are studied under the action of magnetic field and classified into three parts according to following properties.

1. **Magnetic Intensity (Magnetising Force) :** $H = \dfrac{B_0}{\mu_0}$

   B_0 is magnetic field in vacuum

   SI unit = A/m

2. **Intensity of Magnetisation :** Magnetic moment developed/volume

   $I = \dfrac{M}{V}$ (Unit A/m)

$\Rightarrow \quad I = \dfrac{\text{Pole strength}}{\text{area}} \qquad \begin{bmatrix} \because M = m \times l \\ \because V = A \times l \end{bmatrix}$

3. **Magnetic Induction or Magnetic Flux Density (B) :** Number of magnetic field lines crossing per unit area normally through a magnetic substance.

$B = B_0 + \mu_0 I$

$B = \mu_0 H + \mu_0 I$

$B = \mu_0 (H + I)$

$\begin{bmatrix} B_0 = \text{applied magnetic field} \\ \mu_0 I = \text{magnetic field due to magnetisation} \end{bmatrix}$

4. **Magnetic Susceptibility :** $\chi_m = \dfrac{I}{H}$ (no unit)

5. **Magnetic Permeability :** $\mu = \dfrac{B}{H} \Rightarrow B = \mu H$

From above $B = \mu_0(H + I)$

$\Rightarrow \quad \mu H = \mu_0 (H + I)$

$$\frac{\mu}{\mu_0} = 1 + \frac{I}{H}$$

$\mu_r = 1 + \chi_m$ where μ_r = relative permeability.

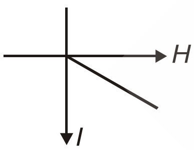
Curie Law

Magnetic susceptibility of paramagnetic material is inversely proportional to its absolute temperature.

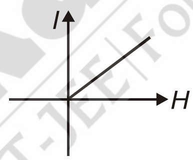
$$\chi_m \propto \frac{1}{T}$$

Variation of *I* with *H*

1. Diamagnetic



2. Paramagnetic

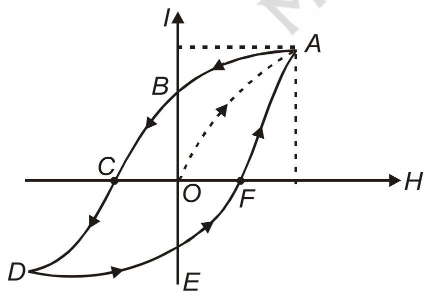


3. Ferromagnetic (Hysteresis)

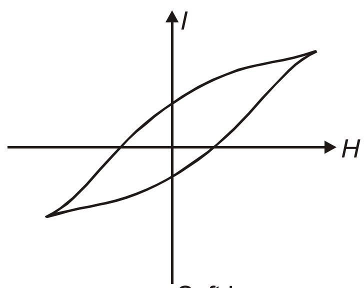
OB = Retentivity (residual magnetism even after magnetising field is reduced to zero)

OC = Coercivity (reverse magnetic field required to reduce residual magnetism to zero)

Area $ABCDEFA$ = Energy loss/cycle during magnetisation and demagnetisation.



Steel
(High coercivity, high retentivity)



Soft iron
(Low coercivity, low retentivity)

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