NCERT Solutions for Class 11 Physics Chapter 13 Kinetic Theory

Q13.1 Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3 Å.

Answer:

The diameter of an oxygen molecule, d = 3 Å.

The actual volume of a mole of oxygen molecules V_{actual} is

The volume occupied by a mole of oxygen gas at STP is V_{molar} = 22.4 litres

$$\frac{V_{actual}}{V_{molar}} = \frac{8.51 \times 10^{-3}}{22.4}$$
$$\frac{V_{actual}}{V_{molar}} = 3.8 \times 10^{-4}$$

Q13.2 Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP: 1 atmospheric pressure, 0^0C). Show that it is $22.4 Litres$.

Answer:

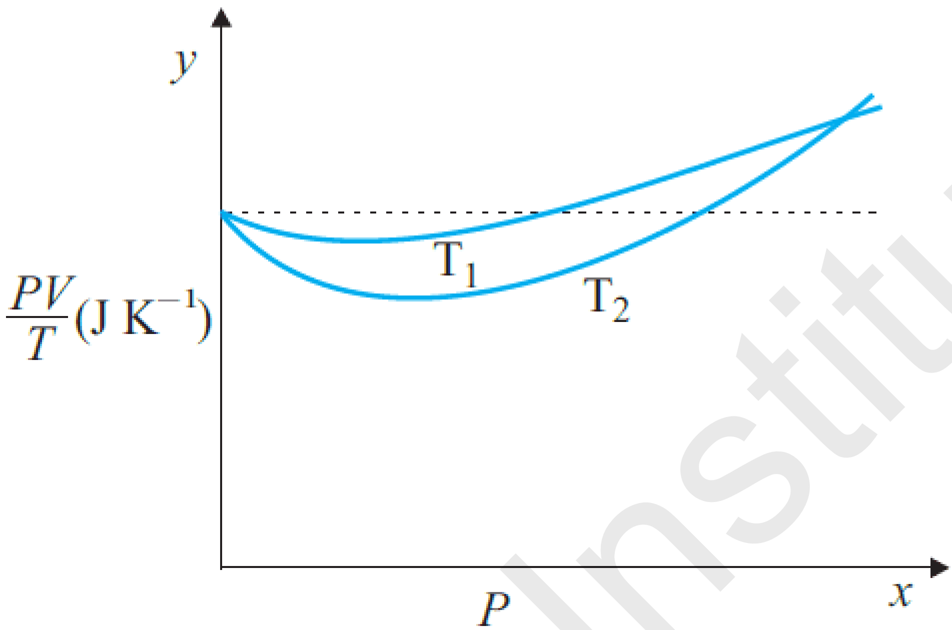
As per the ideal gas equation

$$PV = nRT$$
$$V = \frac{nRT}{P}$$

For one mole of a gas at STP we have

$$V = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5}$$
$$V = 0.0224 m^3$$
$$V = 22.4 \; litres$$

Q13.3 Figure 13.8 shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.



(a) What does the dotted plot signify?

(b) Which is true: $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y-axis?

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ? (Molecular mass of $H_2 = 2.02\mu$, of $O_2 = 32.0\mu$, $R = 8.31 J mol^{-1} K^{-1}$.)

Answer:

(a) The dotted plot corresponds to the ideal gas behaviour.

(b) We know the behaviour of a real gas tends close to that of ideal gas as its temperature increases and since the plot corresponding to temperature T_1 is closer to the horizontal line that the one corresponding to T_2 we conclude T_1 is greater than T_2 .

(c) As per the ideal gas equation

$$\frac{PV}{T} = nR$$

The molar mass of oxygen = 32 g

$$n = \frac{1}{32}$$

R = 8.314

$$nR = \frac{1}{32} \times 8.314$$
$$nR = 0.256 JK^{-1}$$

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen we would not get the same value of PV/T at the point where the curves meet on the y-axis as 1 g of Hydrogen would contain more moles than 1 g of Oxygen because of having smaller molar mass.

Molar Mass of Hydrogen M = 2 g

mass of hydrogen

$$m = \frac{PV}{T}\frac{M}{R} = 0.256 \times \frac{2}{8.314} = 5.48 \times 10^{-5} Kg$$

Q13.4 An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of $27^0 C$. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to $17^0 C$. Estimate the mass of oxygen taken out of the cylinder ($R = 8.31 Jmol^{-1} K^{-1}$, molecular mass of $O_2 = 32 \mu$).

Answer:

Initial volume, V_1 = Volume of Cylinder = 30 l

Initial Pressure P_1 = 15 atm

Initial Temperature T_1 = 27 °C = 300 K

The initial number of moles n_1 inside the cylinder is

Final volume, V_2 = Volume of Cylinder = 30 l

Final Pressure P_2 = 11 atm

Final Temperature T_2 = 17 °C = 290 K

Final number of moles n_2 inside the cylinder is

Moles of oxygen taken out of the cylinder = n_2 -n_1 = 18.28 - 13.86 = 4.42

Mass of oxygen taken out of the cylinder m is

$$m = 4.42 \times 32$$
$$m = 141.44g$$

Q13.5 An air bubble of volume $1.0cm^3$ rises from the bottom of a lake $40m$ deep at a temperature of 12^0C . To what volume does it grow when it reaches the surface, which is at a temperature of 35^0C ?

Answer:

Initial Volume of the bubble, $V_1 = 1.0\ cm^3$

Initial temperature, $T_1 = 12\ ^oC = 273 + 12 = 285\ K$

The density of water is $\rho_w = 10^3\ kg\ m^{-3}$

Initial Pressure is P_1

Depth of the bottom of the lake = 40 m

Final Temperature, $T_2 = 35\ ^oC = 35 + 273 = 308\ K$

Final Pressure = Atmospheric Pressure $= 1.013 \times 10^5 Pa$

Let the final volume be V_2

As the number of moles inside the bubble remains constant we have

Q13.6 Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity $25.0m^3$ at a temperature of 27^0C and $1atm$ pressure.

Answer:

The volume of the room, $V = 25.0\ m^3$

Temperature of the room, $T = 27\ ^oC = 300\ K$

The pressure inside the room, P = 1 atm

Let the number of moles of air molecules inside the room be n

$$n = \frac{PV}{RT}$$
$$n = \frac{1.013 \times 10^5 \times 25}{8.314 \times 300}$$
$$n = 1015.35$$

Avogadro's Number, $N_A = 6.022 \times 10^{23}$

Number of molecules inside the room is N

$$N = nN_A$$
$$N = 1015.35 \times 6.022 \times 10^{23}$$
$$N = 6.114 \times 10^{26}$$

Q13.7 Estimate the average thermal energy of a helium atom at (i) room temperature ($27^0 C$), (ii) the temperature on the surface of the Sun ($6000K$), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).

Answer:

The average energy of a Helium atom is given as $\frac{3kT}{2}$ since it is monoatomic

(i)

$$E = \frac{3kT}{2}$$
$$E = \frac{3 \times 1.38 \times 10^{-23} \times 300}{2}$$
$$E = 6.21 \times 10^{-21}\ J$$

(ii)

$$E = \frac{3kT}{2}$$
$$E = \frac{3 \times 1.38 \times 10^{-23} \times 6000}{2}$$
$$E = 1.242 \times 10^{-19}\ J$$

(iii)

$$E = \frac{3kT}{2}$$
$$E = \frac{3 \times 1.38 \times 10^{-23} \times 10^{7}}{2}$$
$$E = 2.07 \times 10^{-16}\ J$$

Q13.8 Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Answer:

As per Avogadro's Hypothesis under similar conditions of temperature and pressure equal volumes of gases contain equal number of molecules. Since the volume of the vessels are the same and all vessels are kept at the same conditions of pressure and temperature they would contain equal number of molecules.

Root mean square velocity is given as

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

As we can see v_{rms} is inversely proportional to the square root of the molar mass the root mean square velocity will be maximum in case of Neon as its molar mass is the least.

Q13.9 At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20^0C ? (atomic mass of $Ar = 39.9\mu$, of $He = 4.0\mu$).

Answer:

As we know root mean square velocity is given as $v_{rms} = \sqrt{\frac{3RT}{M}}$

Let at temperature T the root mean square speed of an atom in an argon cylinder equal to the rms speed of a helium gas atom at -20^0C

$$\sqrt{\frac{3R \times T}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$$
$$T = 2523.7\ K$$

Q13.10 Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at $2.0atm$ and temperature 17^0C . Take the radius of a nitrogen molecule to be roughly $1.0A$. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of $N_2 = 28.0\mu$).

Answer:

Pressure, P = 2atm

Temperature, T = 17 °C

The radius of the Nitrogen molecule , $r = 1\ \mathring{A}$

The molecular mass of N_2 = 28 u

The molar mass of N_2 = 28 g

From ideal gas equation

$$PV = nRT$$
$$\frac{n}{V} = \frac{P}{RT}$$

The above tells us about the number of moles per unit volume, the number of molecules per unit volume would be given as

The mean free path λ is given as

The root mean square velocity v_{rms} is given as

The time between collisions T is given as

Collision time T' is equal average time taken by a molecule to travel a distance equal to its diameter

$$T' = \frac{d}{v_{rms}}$$
$$T' = \frac{2 \times 1 \times 10^{-10}}{508.26}$$
$$T' = 3.935 \times 10^{-13} s$$

The ratio of the average time between collisions to the collision time is

$$\frac{T}{T'} = \frac{2.18 \times 10^{-10}}{3.935 \times 10^{-13}}$$
$$\frac{T}{T'} = 554$$

Thus we can see time between collisions is much larger than the collision time.

NCERT solutions for class 11 physics chapter 13 kinetic theory additional exercise

Q13.11 A metre long narrow bore held horizontally (and closed at one end) contains a $76cm$ long mercury thread, which traps a $15cm$ column of air. What happens if the tube is held vertically with the open end at the bottom?

Answer:

Initially, the pressure of the 15 cm long air column is equal to the atmospheric pressure, P_1 = 1 atm = 76 cm of Mercury

Let the crossectional area of the tube be x cm^2

The initial volume of the air column, V_1 = 15x cm^3

Let's assume once the tube is held vertical y cm of Mercury flows out of it.

The pressure of the air column after y cm of Mercury has flown out of the column P_2 = 76 - (76 - y) cm of Mercury = y cm of mercury

Final volume of air column V_2 = (24 + y)x cm^3

Since the temperature of the air column does not change

$$P_1V_1 = P_2V_2$$
$$76 \times 15x = y \times (24 + y)x$$
$$1140 = y^2 + 24y$$
$$y^2 + 24y - 1140 = 0$$

Solving the above quadratic equation we get y = 23.8 cm or y = -47.8 cm

Since a negative amount of mercury cannot flow out of the column, y cannot be negative.

Therefore y = 23.8 cm.

Length of the air column = y + 24 = 47.8 cm.

Therefore once the tube is held vertically, 23.8 cm of Mercury flows out of it and the length of the air column becomes 47.8 cm

Q 13.12 From a certain apparatus, the diffusion rate of hydrogen has an average value of $28.7 cm^3 s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of $7.2 cm^3 s^{-1}$. Identify the gas.

Answer:

As per Graham's Law of diffusion if two gases of Molar Mass M_1 and M_2 diffuse with rates R_1 and R_2 respectively their diffusion rates are related by the following equation

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

In the given question

$R_1 = 28.7\ cm^3\ s^{-1}$

$R_2 = 7.2\ cm^3\ s^{-1}$

$M_1 = 2$ g

The above Molar Mass is close to 32, therefore, the gas is Oxygen.