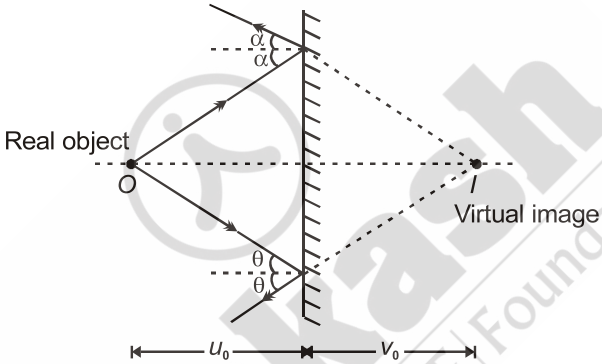
Chapter 15

Optics

PLANE MIRROR

Following are some important points regarding image formation by plane mirror :

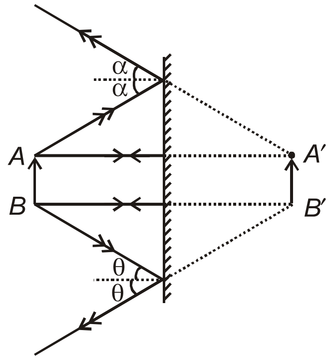


1. Object distance = Image distance, $|u_0| = |v_0|$.

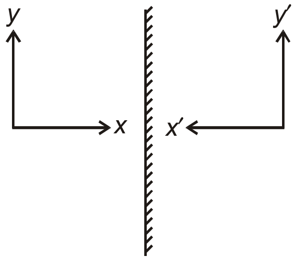
2. $\frac{du}{dt} = -\frac{dv}{dt}$, *i.e.*, speed of object w.r.t. mirror is equal to speed of image w.r.t. mirror. For example, if object is at rest and mirror is moving with velocity x towards object then velocity of image will be $2x$.

3. For extended object, it can be seen that height of object = height of image.

$\Rightarrow$ Magnification = $\frac{\text{Height of image}}{\text{Height of object}} = 1$



4. Image is laterally inverted, i.e., it has front back reversed.



5. Keeping incident ray fixed, if a plane mirror is rotated by an angle θ, reflected ray rotates by an angle 2θ.
6. For two mirrors inclined at an angle 'θ'. Number of images formed by the mirrors for a point object are

   (a) $\frac{360°}{\theta} - 1$, if $\frac{360°}{\theta}$ = even number

   (b) $\frac{360°}{\theta} - 1$, when $\frac{360°}{\theta}$ = odd and object is placed symmetrically.

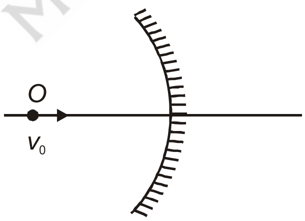
   (c) $\frac{360°}{\theta}$, when $\frac{360°}{\theta}$ = odd and object is placed unsymmetrically.

REFLECTION FROM CURVED SURFACES

1. $f = \frac{R}{2}$
2. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
3. $m = \frac{I}{O} = -\frac{v}{u} = \frac{f-v}{f} = \frac{f}{f-u}$

where,
f = Focal length
u = Object distance
v = Image distance
I = Size of image
O = Size of object

4. Magnification is negative for inverted image and positive for erect image.
5. If object O is moving with velocity v_0 as shown in figure then velocity of image $v_{image} = -m^2 v_0$



$\because \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$

$\Rightarrow \quad v_{image} = -m^2 v_0$

6. If object is moving with velocity v_0 as shown in figure then velocity of real image $v_{image} = mv_0$

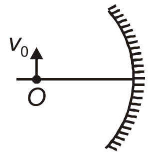
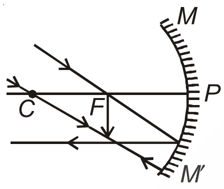
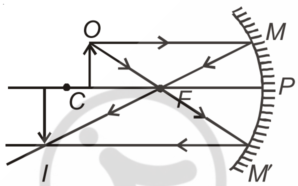
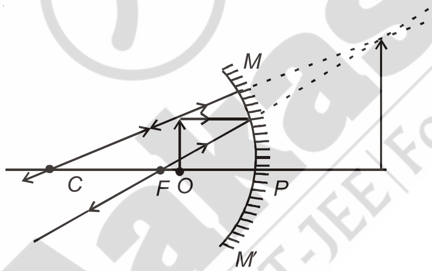


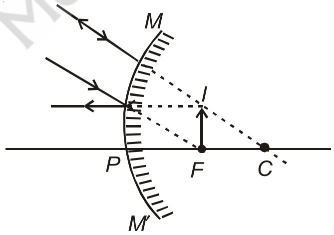
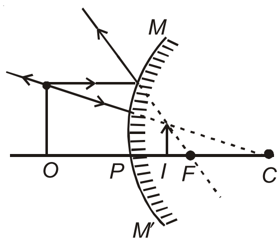
Image formation for Concave Mirror

Table : Position and nature of image for a given position of object

S.No.	Position of object	Ray diagram	Details of image
1.	At infinity		Real inverted, very small [$m \to (0^-)$]; at focus or in focal plane.
2.	Between F and C		Real, inverted, large ($m < -1$) beyond C
3.	Between F and P		Virtual, erect ($m > +1$) Behind the mirror

For Convex Mirror

Table: Position and nature of image for a given position of object

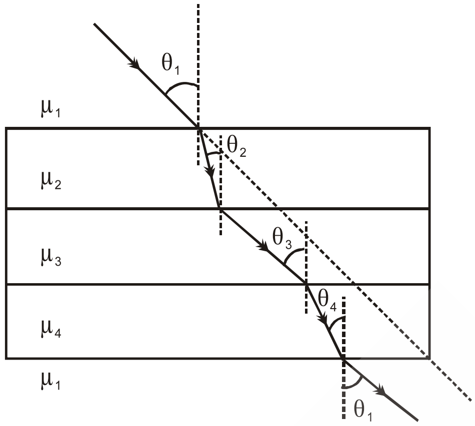
S.No.	Position of object	Ray diagram	Details of image
1.	At infinity		Virtual, erect, very small ($0 < m << +1$) at F
2.	In front of mirror		Virtual, erect, diminished ($m < +1$) between P and F

Laws of Refraction (Snell's Law)

Consider a light ray crossing a series of media as shown.

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 = \mu_3 \sin\theta_3 = \mu_4 \sin\theta_4 = \mu_1 \sin\theta_1$

$\Rightarrow$ $\mu \sin\theta$ = Constant



Incident and refracted are parallel if they are in same medium

REAL DEPTH AND APPARENT DEPTH

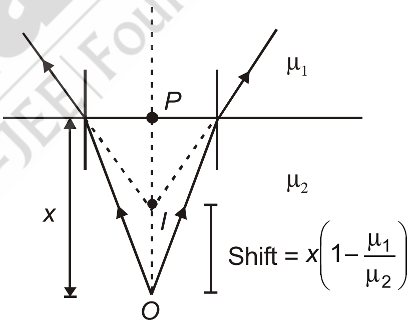
Consider that rays from object O are going from medium 2 to 1 ($\mu_2 > \mu_1$) and image is formed at I.

$\Rightarrow$ $\dfrac{OP}{\mu_2} = \dfrac{PI}{\mu_1}$ (By Snell's law)

$\Rightarrow$ $PI = \dfrac{\mu_1}{\mu_2} OP = \dfrac{\mu_1}{\mu_2} x$

$\because$ $\mu_1 < \mu_2$, $PI < OP$

$\Rightarrow$ Shift $= x\left(1 - \dfrac{\mu_1}{\mu_2}\right)$



$\Rightarrow$ Velocity of image $= \dfrac{\mu_1}{\mu_2}$ (Velocity of object)

Glass-slab

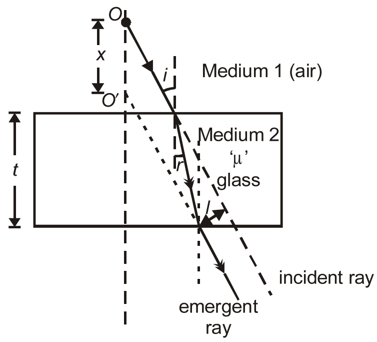
Different results for refraction by glass slab are :

1. Incident and emergent rays are parallel.

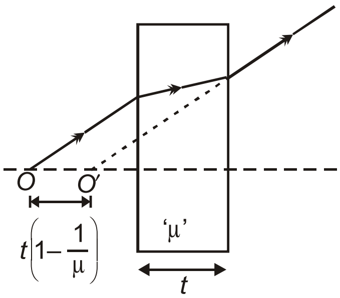
2. l = lateral displacement = $\dfrac{t \sin(i - r)}{\cos r}$

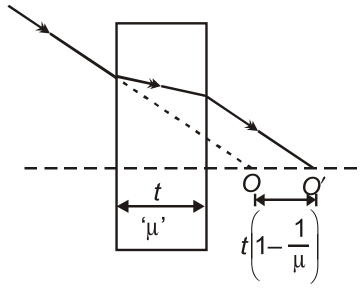
   for small values of i, $\sin(i - r) \approx i - r$ and $r \to 0$.

   $\Rightarrow$ $l = t\left[i - \dfrac{i}{\mu}\right] = ti\left[1 - \dfrac{1}{\mu}\right]$

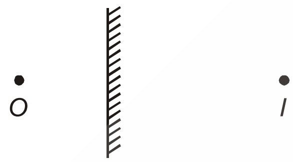


3. $I \to I_{max}$, when $i \to 90°$

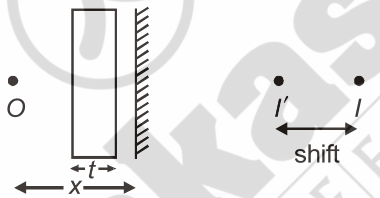
4. 

5. 

If an object is placed at distance x from plane mirror then image will be formed at distance of x from mirror.



Now if a glass slab of thickness t is introduced between object and mirror then image will shift toward object by $2t\left(1-\frac{1}{\mu}\right)$



Critical Angle

If a ray is travelling from optically denser medium to optically rarer medium, then critical angle may be defined as the angle of incidence in denser medium corresponding to which angle of refraction in rarer medium is 90°.

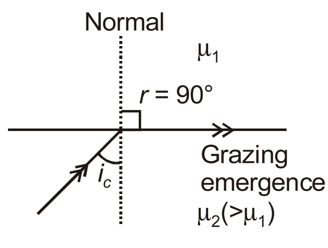
If μ_1 = refractive index of rarer medium

μ_2 = refractive index of denser medium

and i_c = critical angle

then $\mu_2 \sin i_c = \mu_1 \sin 90°$

$\Rightarrow \quad \sin i_c = \frac{\mu_1}{\mu_2}$

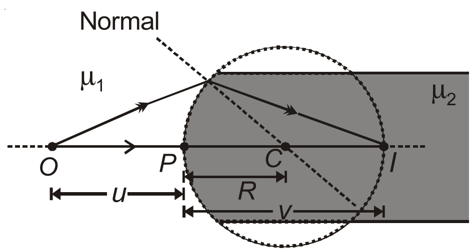


Spherical Refracting Surface

For object O, image formed by refracting surface is at I.

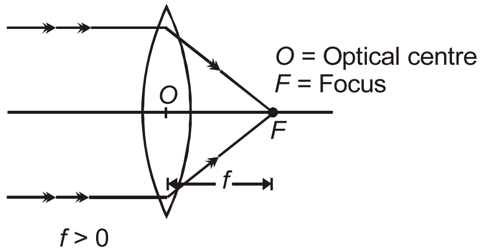
Object distance (u) and image distance (v) are related as

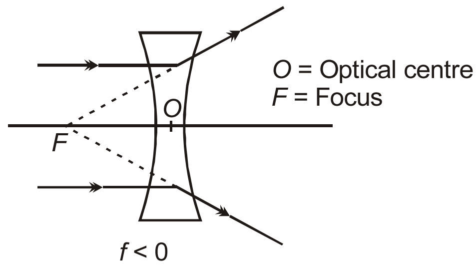
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$



Some Important Relations for Lenses

Converging (Convex) lens **Diverging (Concave) lens**



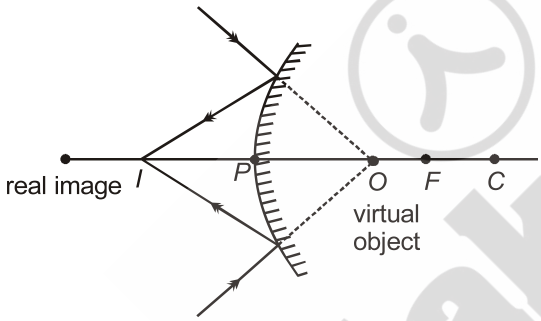


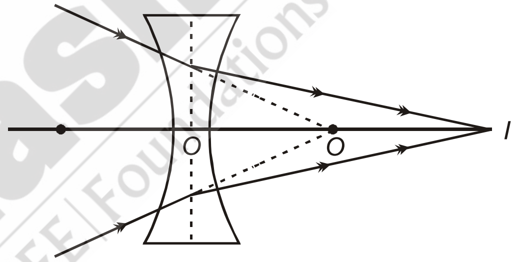
1. Lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

2. Linear magnification $m=\frac{I}{O}$ $m<0$ for inverted image; $m>0$ for erect image

3. In lenses, $m=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$

4. A convex mirror or a concave lens can form a real image if object is virtual as shown.





5. If an object moves along the axis of a convex lens from infinity towards its focus with a constant speed, then $v_i = m^2 v_0$, for a lens $v_i = \frac{v^2}{u^2}v_0$

6. If an object moves perpendicular to axis of a convex lens with a velocity v_0 then $v_i = mv_0$

LATERAL MAGNIFICATION

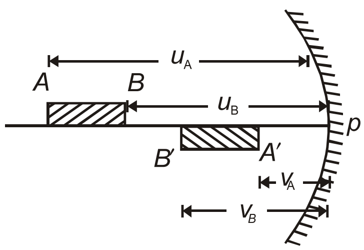
$$m_L=\frac{A'B'}{AB}=\frac{v_B-v_A}{u_A-u_B}$$

For short object

$$m_L=\frac{dv}{du}$$

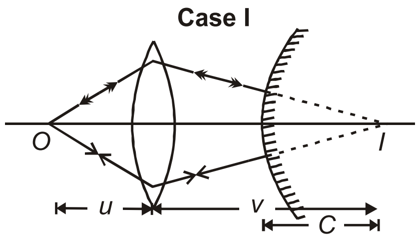
For a concave mirror, $m_L=-\frac{v^2}{u^2}$

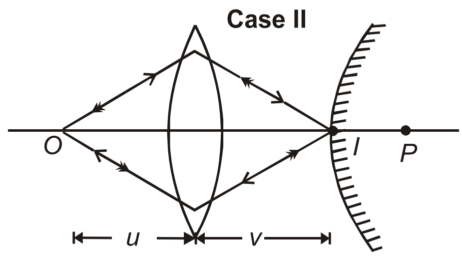
For a convex lens, $m_L=\frac{v^2}{u^2}$



Lens and Mirror

A point object is placed in front of a convex lens. A convex mirror is placed behind the lens, so that final image coincides with the object itself.



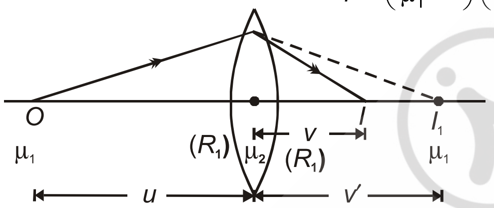


Final image is real and inverted and coincides with O

Final image is real and erect and coincides with O

Lens Maker's Formula (For thin lenses)

For a thin lens, $P(\text{Power}) = \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

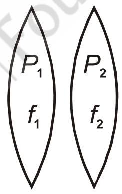


COMBINATION OF LENSES

1. In contact :

$P = P_1 + P_2$, P is taken with sign

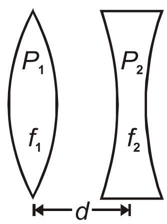
$\Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$



2. $P = P_1 + P_2 - dP_1P_2$

$\Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ (Applicable only for parallel rays)

where d is the separation between two lenses in air

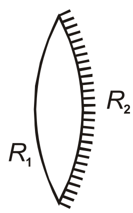


Silvering of Lenses

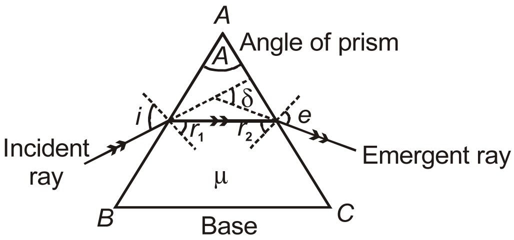
1. When a convex lens is silvered it will behave like a concave mirror.
2. When a concave lens is silvered it will behave like a convex mirror.
3. **Case-1 :** When one face (of radius of curvature R_2) of a double convex lens is silvered

$P_{eq} = 2P_l + P_m$

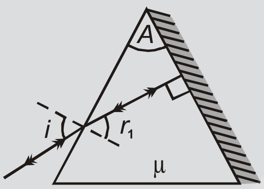
$\Rightarrow \frac{1}{F_{eff}} = 2(\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] - \frac{2}{R_2}$



PRISM AND DISPERSION



Note :

1. *The variation of δ with i is unsymmetrical.*
2. *Under minimum deviation, ray passes symmetrically through the prism.*
3. *If prism is isosceles or equilateral, refracted ray is parallel to base of prism under minimum deviation.*
4. *If $A > 2C$ or $\mu > \text{cosec}\,\dfrac{A}{2}$, there will be no emergent light whatever may be the angle of incidence.*
5. *$A = 2C$ is called limiting value of angle of prism.*
6. *If $A < C$, total internal reflection at second face can never take place.*
7. 

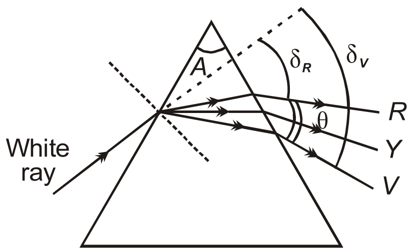
   In case, one face is silvered, for incident ray to retrace its path after reflection from 2nd face.

   $r_2 = 0 \qquad r_1 = A$

   $\Rightarrow \quad \mu = \dfrac{\sin i}{\sin A}$
8. *Angle of deviation is maximum when angle of incidence = 90º.*
9. *A thin hollow prism as shown produces zero deviation.*

Dispersion

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \frac{D}{\lambda^6} + \ldots \quad \text{[Cauchy's formula]}$$



$\delta = (\mu - 1)A$

As, $\lambda_V < \lambda_R$

$\Rightarrow \mu_V > \mu_R$

$\Rightarrow \delta_V < \delta_R$

θ = angular dispersion = $\delta_V - \delta_R = (\mu_V - \mu_R)A$

δ = Mean deviation = $(\mu - 1)A$

μ = mean refractive index = $\dfrac{\mu_V + \mu_R}{2}$

Dispersive power $\omega = \dfrac{\theta}{\delta} = \dfrac{(\mu_V - \mu_R)A}{(\mu - 1)A} = \dfrac{\mu_V - \mu_R}{\mu - 1} = \dfrac{d\mu}{\mu - 1}$

$d\mu$ = difference in refractive index

Combination of Prisms

$\theta = \theta_1 + \theta_2$, $\delta = \delta_1 + \delta_2$

Case-1 : For dispersion without deviation

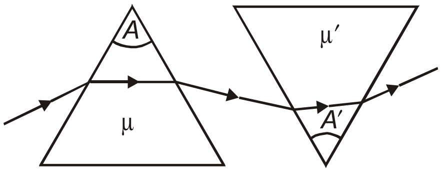
$\delta = 0$

$$\Rightarrow \quad (\mu - 1)A + (\mu' - 1)A' = 0$$

Case-2 : For deviation without dispersion

$\theta = 0$

$$\Rightarrow \quad (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$$



Chromatic Aberration

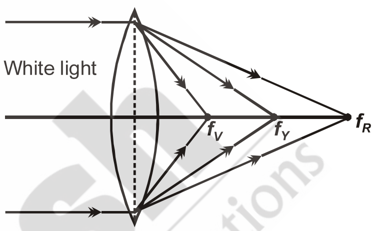
This is the phenomena observed when different colours come to focus at different points on the principal axis. This defect can be removed by using an achromatic lens combination.

As, $\mu = a + \dfrac{b}{\lambda^2} + \dfrac{c}{\lambda^4}$

and $\lambda_R > \lambda_Y > \lambda_v$

$\Rightarrow \mu_R < \mu_Y < \mu_v$

$\Rightarrow f_R > f_Y > f_v$ $\quad \left[\text{as } \dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\right]$



Achromatic Combination

A combination free from chromatic aberration is achromatic combination.

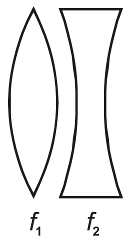
1. Lenses in contact

Power $P = \dfrac{1}{f_1} + \dfrac{1}{f_2} = P_1 + P_2$

Condition for achromatism

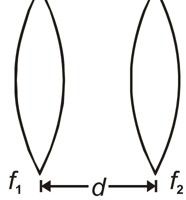
$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{or} \quad \omega_1 P_1 + \omega_2 P_2 = 0$$

$\Rightarrow$ P_1 and P_2 or f_1 and f_2 should be opposite sign also $\omega_1 \neq \omega_2$ as P will become zero.



2. $P = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2} = P_1 + P_2 - dP_1 P_2$

Condition for achromatism $d = \dfrac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$.



Note : *(a) f_1 and f_2 can be of same or opposite sign*

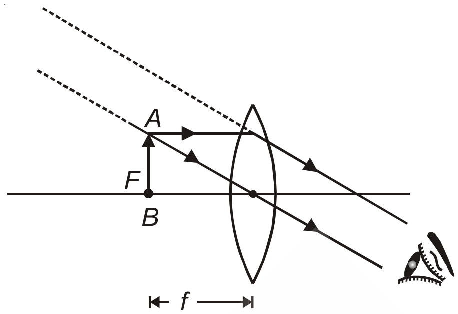
(b) $\omega_1 = \omega_2 \Rightarrow d = \dfrac{f_1 + f_2}{2}$

OPTICAL INSTRUMENTS

1. **Simple Microscope / Magnifying Glass :** It uses a single convex lens of focal length f

$m = \frac{D}{u}$, where u is distance of object

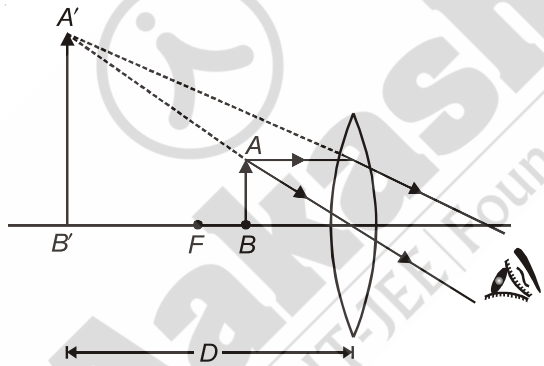
Case-1 :



(Final image at infinity)

For relaxed eye, $u = f$, image at $\infty \Rightarrow m = \frac{D}{f} > 0$

Case-2 :



(Final image at least distance of distinct vision)

For strained eye, image is at $D \Rightarrow m = 1+\frac{D}{f} > 0$

2. **Compound Microscope**

It uses two convex lens objective (f_o) and eyepiece (f_e)

u_o = object distance from objective (u_o is close to f_o)

v_o = image distance from objective (close to length of tube)

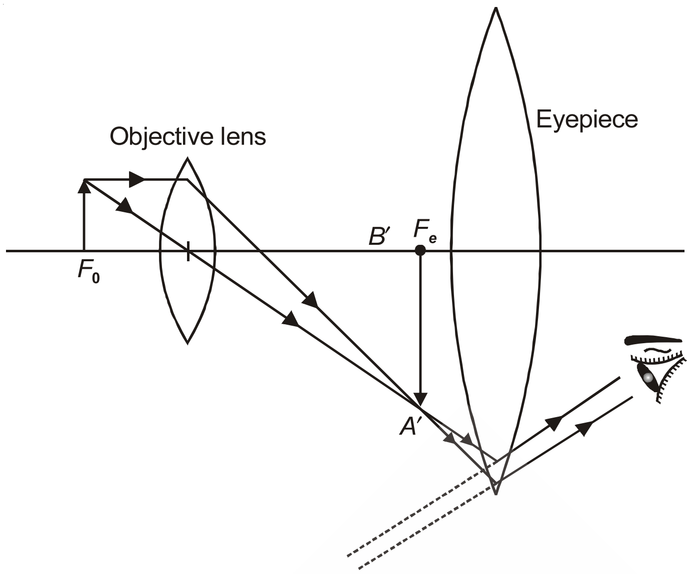
Magnification by objective $m_o = -\frac{v_o}{u_o}$ (–ve)

Magnification by eyepiece $m_e = \frac{D}{u_e}$

Magnification for microscope

$m = m_o \times m_e = \frac{-v_o}{u_o} \times \frac{D}{u_e}$

Case-1 :

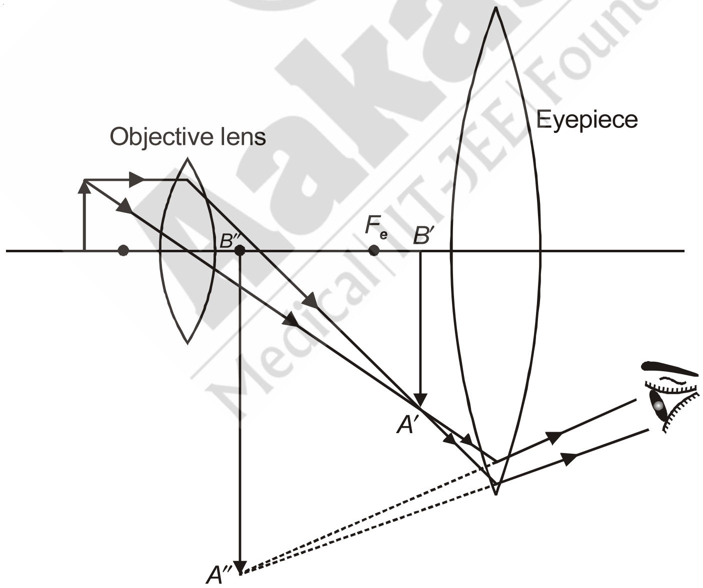


(Final image at infinity)

Relaxed eye : $m = \frac{-v_o}{u_o} \times \frac{D}{f_e} \approx \frac{-L}{f_o}\left(\frac{D}{f_e}\right)$

Length of tube $L = V_0 + f_e$

Case-2 :



(Final image at least distance of distinct vision)

Strained eye : $m = \frac{-v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \approx -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$

Length of tube $V_0 + \frac{f_e D}{f_e + D}$

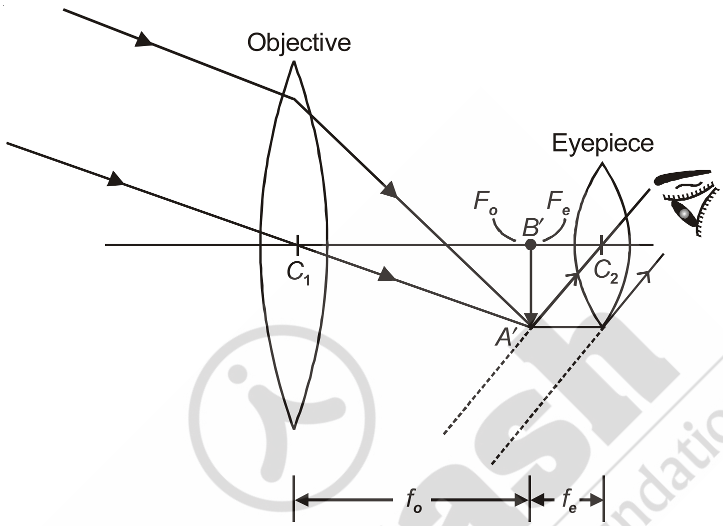
3. **Astronomical Telescope :** f_o is focal length of objective and f_e is focal length of eye-piece.

$m = m_o \times m_e$ here $m_o < 0$, $m_e > 0$., $m < 0$

Case-1 :

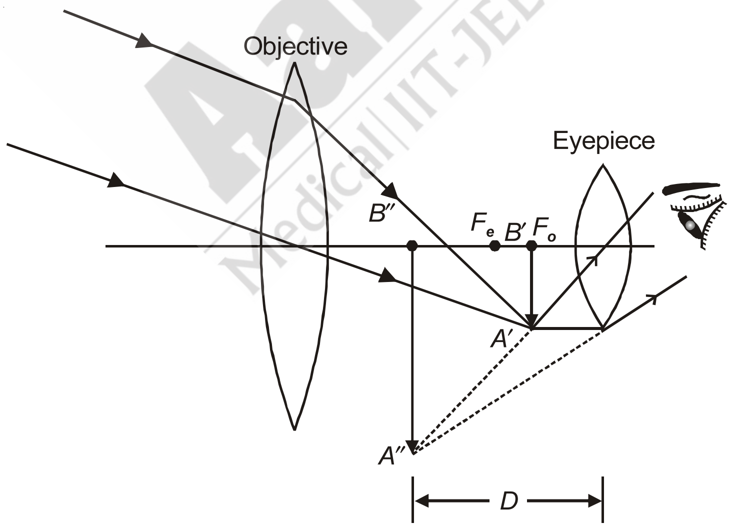
For relaxed eye *i.e.,* normal adjustment. $m = -\frac{f_o}{f_e}$

Length of tube $L = f_o + f_e$



(Final image at infinity)

Case-2 :



(Final image at least distance of distinct vision)

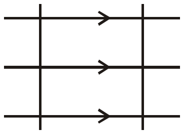
For strained eye : $m = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

length of tube $L < f_o + f_e$

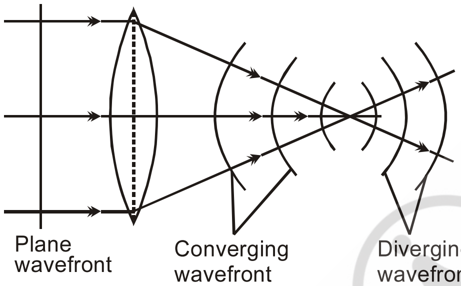
WAVE OPTICS

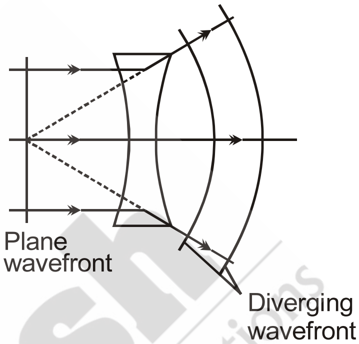
Plane Wavefront (Plane sheet source)

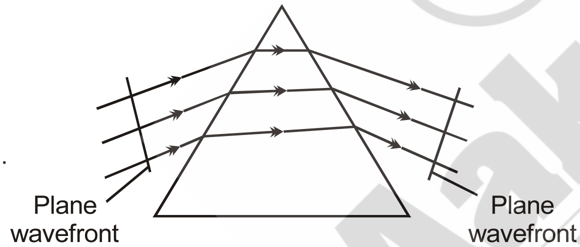
1. Amplitude = constant $\Rightarrow$ $A \propto r^0$
2. Intensity = constant $\Rightarrow$ $I \propto r^0$



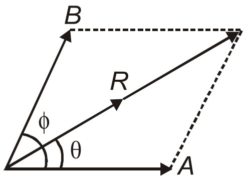
Refraction in form of Wavefronts

1. 

2. 

3. 

SUPERPOSITION OF WAVES



Wave-1 : $y_1 = A \sin \omega t$

Wave-2 : $y_2 = B \sin (\omega t + \phi)$

Resultant wave : $y = y_1 + y_2 \Rightarrow y = A \sin \omega t + B \sin (\omega t + \phi)$

$\Rightarrow y = R \sin (\omega t + \theta)$

where $\tan\theta = \dfrac{B\sin\phi}{A + B\cos\phi}$

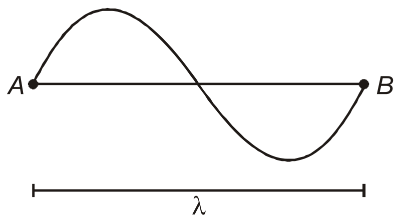
and $R = \sqrt{A^2 + B^2 + 2AB\cos\phi}$

Intensity $\propto$ (Amp)^2 $\therefore$ $I_1 \propto A^2$, $I_2 \propto B^2$, $I \propto R^2$

As, $R^2 = A^2 + B^2 + 2AB \cos \phi \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

Relation between phase difference and path difference

For the two points A and B on a wave,



$\phi_{AB} = 2\pi$

$\Delta X_{AB} = \lambda$

$$\boxed{\phi = \frac{2\pi}{\lambda}\Delta x}$$

Condition for Maxima

When $\cos\phi = 1$ or $\phi = 2n\pi$ or $\Delta x = n\lambda$ (path difference)

$R_{max} = A + B$, $I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

Condition for Minima

When $\cos\phi = -1$ or $\phi = (2n-1)\pi$ or $\Delta x = (2n-1)\dfrac{\lambda}{2}$

$R_{min} = A - B$, $I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$

$\Rightarrow \dfrac{R_{max}}{R_{min}} = \dfrac{A+B}{A-B}$ $\qquad \dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{\beta}+1}{\sqrt{\beta}-1}\right)^2$, where $\beta = \dfrac{I_1}{I_2}$

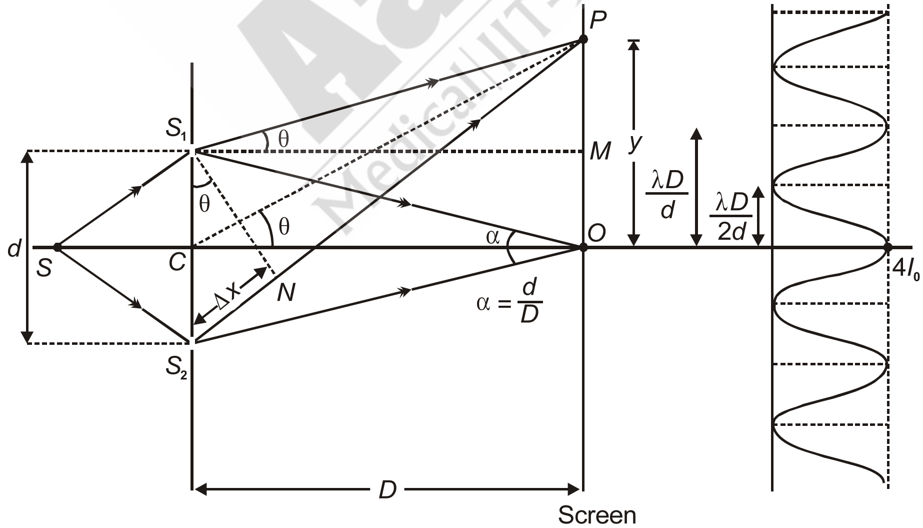
Interference

Phenomenon of redistribution of energy on account of superposition of waves is known as interference.

Coherent Sources

Condition for sustained interference-Sources must be coherent *i.e.*, phase difference between them must be constant.

Young's Double Slit Experiment



For waves reaching P from S_1 and S_2, path difference $\Delta x = d\sin\theta$.

For small θ, $\Delta x = \dfrac{yd}{D}$ $\qquad \left[\because\ \sin\theta \approx \tan\theta = \dfrac{y}{D}\right]$

Condition for Maxima

For maxima $\Delta x = n\lambda \Rightarrow y = n\dfrac{\lambda D}{d}$ $(n = 0, 1, 2, \ldots)$

or $d \sin\theta = n\lambda$

$\Rightarrow \sin\theta = \dfrac{n\lambda}{d}$

Condition for Minima

For minima $\Delta x = (2n-1)\dfrac{\lambda}{2} \Rightarrow y = \dfrac{(2n-1)\lambda D}{2d}$ $(n = 1, 2, \ldots)$

or $d\sin\theta = \dfrac{(2n-1)\lambda}{2}$

$\Rightarrow \sin\theta = \dfrac{(2n-1)\lambda}{2d}$

If λ and d are comparable then, as $-1 \le \sin\theta \le 1$

$-1 \le \dfrac{(2n-1)\lambda}{2d} \le 1$

$\Rightarrow \dfrac{-2d+\lambda}{2\lambda} \le n \le \dfrac{2d+\lambda}{2\lambda}$

From here you can find maximum number of dark fringes observed on the screen.

For example, if $d = 2\lambda$, then

$\dfrac{-4\lambda+\lambda}{2\lambda} \le n \le \dfrac{4\lambda+\lambda}{2\lambda}$

$-\dfrac{3}{2} \le n \le \dfrac{5}{2}$ so, n can have four values *i.e.*, –1, 0, 1, 2.

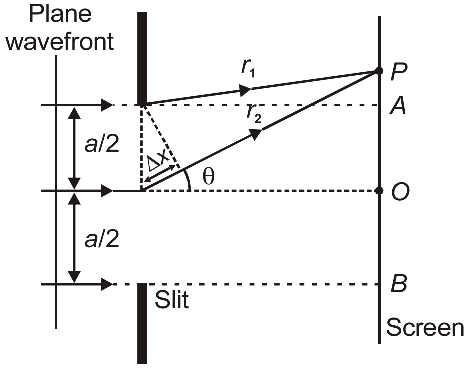
This means that only four minima are observed on the screen.

DIFFRACTION

Diffraction is the phenomenon of light observed due to superposition of secondary wavelets starting from different points of a wavefront which is not blocked by an obstacle or which are allowed by an aperture (of size comparable to the wavelength of light).

In other words you can say diffraction is the phenomena of entering of light in the region of geometrical shadow, due to bending around obstacle edges.

Diffraction by Single Slit



1. $\Delta x = \frac{a}{2}\sin\theta$
2. For '*O*' waves from all points in the slit travel about the same distance and are in phase.
3. At *P*, waves r_1 and r_2 have a phase difference $\Delta x = \frac{a}{2}\sin\theta$.
4. When $\frac{a}{2}\sin\theta_1 = \frac{\lambda}{2}$, there will be destructive interference.

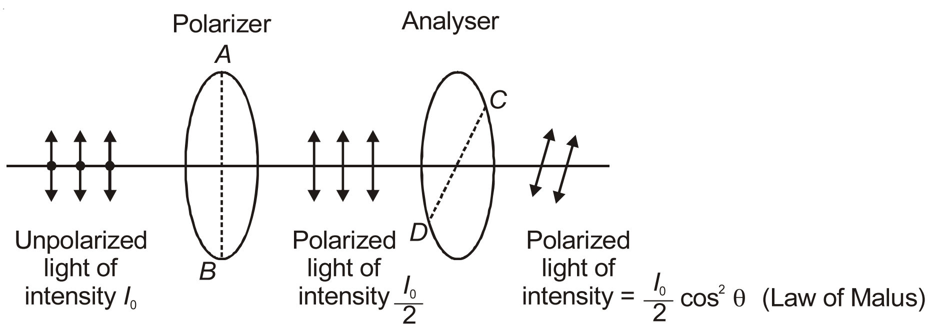
   $\therefore$ when $a \sin \theta_1 = \lambda$, first minima will be formed at *P*.
5. In general $a \sin \theta_n = n\lambda$ is position of n^{th} minima.
6. Angular position of first minima $\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right)$
7. Angular spread of central maximum is $2\theta_1 = 2\sin^{-1}\left(\frac{\lambda}{a}\right)$. If $\lambda << a$, then angular spread = $\frac{2\lambda}{a}$.
8. When $\lambda > a \Rightarrow \sin\theta > 1$ which is not possible

   $\therefore$ diffraction cannot be observed.
9. $\lambda < < a$, then $\sin\theta \approx \theta = \frac{\lambda}{a}$ (in radians)
10. Width of central maximum = $\frac{2\lambda D}{a}$
11. Width of other fringes = $\frac{\lambda D}{a}$.
12. If I_0 is the intensity of central maximum, then intensity of n^{th} maxima is $I_n = \frac{4I_0}{(2n+1)^2\pi^2}$

    $\therefore$ $I_0 : I_1 : I_2 : : 1 : 0.045 : 0 : 016$.
13. The intensity of fringe goes on decreasing in case of diffraction while it remain nearly same in the interference.

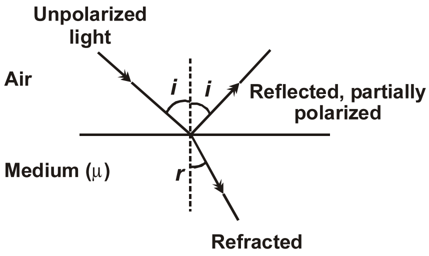
Polarization

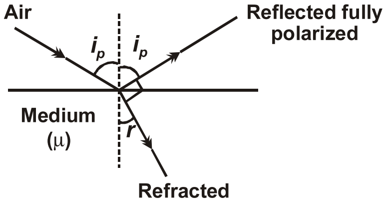
Polarization is a phenomenon exhibited by transverse waves only.



where θ = angle between the transmission (*i.e.*, *AB* and *CD*) axis of the two polaroids

Polarization by Reflection





SOME IMPORTANT POINTS :

1. Reflected light is partially polarized.
2. When $i = i_p$ (polarizing angle), reflected light is completely polarized, i_p is also called Brewster's angle.
3. When reflected light is completely polarized, reflected and refracted light are perpendicular to each other.
4. This was found experimentally by **Sir David Brewster**.

   At this situation $i_p + r = 90°$

   $\Rightarrow$ $\mu = \tan i_p$. This is called Brewster's law.

   $$\Rightarrow \quad \tan i_p = \frac{1}{\sin i_c}$$

   $\Rightarrow$ $\sin i_c . \tan i_p = 1$ [Here i_c is critical angle]

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