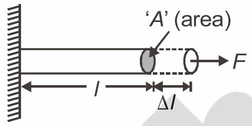
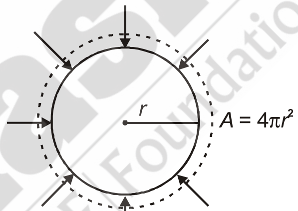
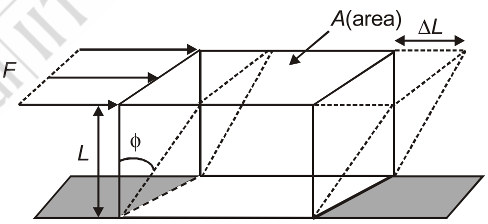
Chapter 7

Properties of Solids and Liquids

Strain	Stress
(1) Longitudinal strain = $\frac{\Delta l}{l}$	(1)  Normal Stress (Tensile) = F/A
(2) Volumetric strain = $-\frac{\Delta V}{V}$	(2)  Normal Stress (Compressive) = P (pressure)
(3) Shear strain = $\phi = \frac{\Delta L}{L}$	(3)  Tangential Stress or Shear Stress = $\frac{F}{A}$

MODULI OF ELASTICITY

(1) Young's modulus of elasticity $(Y) = \frac{\text{Tensile stress}}{\text{Longitudinal strain}} = \frac{Fl}{A\Delta l}$

(2) Bulk modulus of elasticity $(\beta) = \frac{\text{Normal or compressive stress}}{\text{Volumetric strain}} = -V\frac{\Delta P}{\Delta V}$ or, $\beta = -V\frac{dP}{dV}$

Compressibility = $\frac{1}{\beta}$

(3) Modulus of rigidity or shear modulus η or $G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F}{A\phi} = \frac{FL}{A\Delta L}$

Applications :

1. For a wire $Y = \dfrac{Fl}{A\Delta l} \Rightarrow F = \dfrac{YA}{l}\Delta l$

   i.e. a wire behaves like a spring of spring constant (k)

   $$k = \frac{YA}{l} \qquad \left(i.e., k \propto \frac{1}{l}\right)$$

2. When this wire is stretched by applying an external force F, and Δl is extension produced, then

   (a) Work done by external force = $F\Delta l$

   (b) Work done by restoring force = $\dfrac{1}{2}F\Delta l$

   (c) Heat produced = $\dfrac{1}{2}F\Delta l$

   (d) Elastic potential energy stored = $\dfrac{1}{2}F\Delta l$

   Energy density $U = \dfrac{1}{2}\dfrac{F\Delta l}{\text{volume}} = \dfrac{1}{2}\dfrac{F\Delta l}{Al}$

   $$= \frac{1}{2} \text{ stress} \times \text{strain}$$

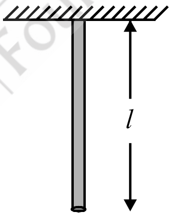
   $$= \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}Y(\text{strain})^2$$

3. A rod of mass m and length l hangs from a support

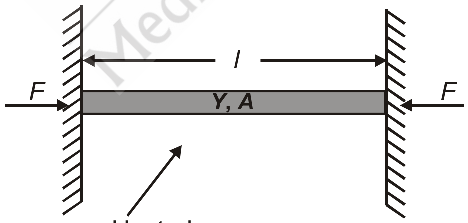
   Area of cross-section = A

   Extension produced due to its own weight,

   $$\Delta l = \frac{Mgl}{2AY} = \frac{\rho g l^2}{2Y} \quad (\rho = \text{density of wire})$$



Thermal Stress : Rod Fixed between Rigid Support



If $\Delta\theta$ = Rise in temperature

Compressive strain = $\dfrac{\Delta l}{l} = \alpha\Delta\theta$

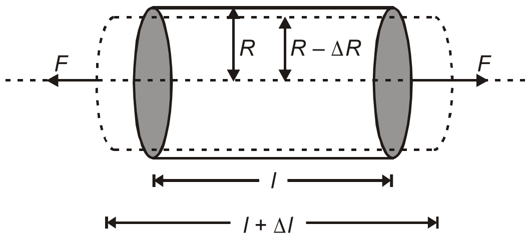
Compressive stress = Y × strain = $Y\alpha\Delta\theta$

$\Rightarrow \quad F = Y\alpha\Delta\theta \times A$

Note : *If the rod is placed on horizontal frictionless surface, then stress developed on heating is zero.*

Poisson's Ratio

Consider a uniform bar being stretched by applying two forces at its ends.



Longitudinal strain = $\frac{\Delta l}{l}$

Lateral strain = $-\frac{\Delta R}{R}$

Poisson's ratio, $\sigma = -\frac{\Delta R / R}{\Delta l / l}$

(a) Theoretically $-1 \le \sigma \le 0.5$

(b) Practically $0 \le \sigma \le 0.5$

(c) When density of material is constant $\Rightarrow \sigma = 0.5$

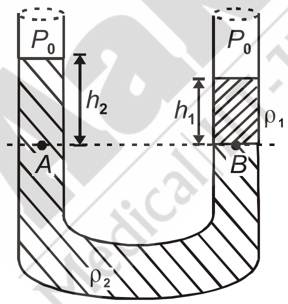
Equilibrium of Different Liquids in a U tube

1. $P_A = P_B$ (as A & B are at same level in same liquid)

   $\Rightarrow P_0 + h_1\rho_1 g = P_0 + h_2\rho_2 g$ (where P_0 is atmospheric pressure)

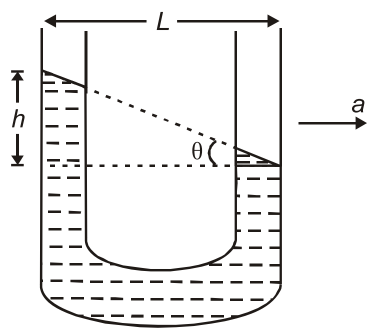
   $\Rightarrow h_1\rho_1 g = h_2\rho_2 g$

   $\Rightarrow \rho_1 h_1 = \rho_2 h_2$

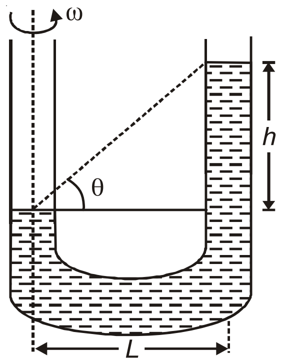


2. When the U tube accelerates horizontally, difference of levels of liquid satisfies the relation,

   $\tan\theta = \frac{a}{g} = \frac{h}{L}$



3. When U-tube is rotated about on limb



Here $h = \frac{\omega^2 L^2}{2g}$ and $\tan\theta = \frac{h}{L} \Rightarrow \tan\theta = \frac{\omega^2 L}{2g}$.

Excess pressure

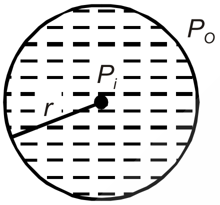
If P_o = Atmospheric pressure

P_i = Inside pressure

then $P_i - P_o$ = Excess pressure

Liquid drop

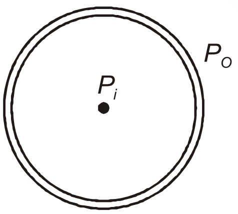
(1)



$$P_i = P_o + \frac{2T}{r}$$

Soap bubble

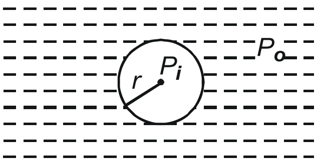
(2)



$$P_i - P_o = \frac{4T}{r}$$

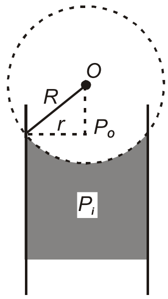
Air bubble

(3)



$$P_i = P_o + \frac{2T}{r}$$

(4) Capillary tube, concave meniscus

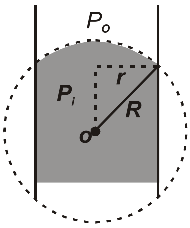


Capillary tube, Concave Meniscus

(a) $P_i = P_o - \frac{2T}{R}$

(b) $F_a > \frac{F_c}{\sqrt{2}}$

(5) Capillary tube, convex meniscus.



Convex Meniscus

(a) $P_i = P_o + \frac{2T}{R}$

(b) $F_a < \frac{F_c}{\sqrt{2}}$

Combining of Bubbles

1. If the soap bubble coalesce in vacuum, then $P_o = 0$

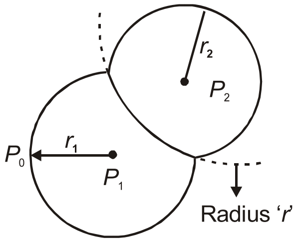
$\Rightarrow \quad r^2 = r_1^2 + r_2^2$

2. If two soap bubbles come in contact to form a double bubble then

r = radius of interface, $r_1 > r_2$

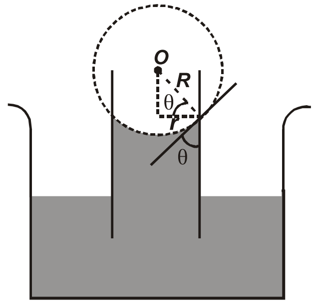
$$\frac{1}{r} = \frac{1}{r_2} - \frac{1}{r_1}$$

The interface will be convex towards larger bubble and concave towards smaller bubble because $P_2 > P_1 > P_0$.



CAPILLARY ACTION

Rise or fall of liquid in a tube of fine diameter.



Ascent formula

$$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta}{r\rho g}$$

Energy of a Liquid

Various energies per unit mass :

1. Potential energy/mass = gh
2. Kinetic energy/mass = $\frac{1}{2}v^2$
3. Pressure energy/mass = $\frac{P}{\rho}$

Energy Heads

Various energy heads per unit mass :

1. Gravitational head = h
2. Velocity head = $\frac{v^2}{2g}$
3. Pressure head = $\frac{P}{\rho g}$

BERNOULLI'S THEOREM

It is based on conservation of energy.

For an ideal, non-viscous and incompressible liquid,

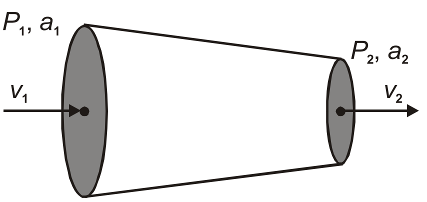
$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + gh_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2} + gh_2 = \text{constant}$$

Applications of Bernoulli's Theorem

(1) To find rate of flow of liquid $Q = av$. Value of Q in various cases has been given below

Case - (a) :

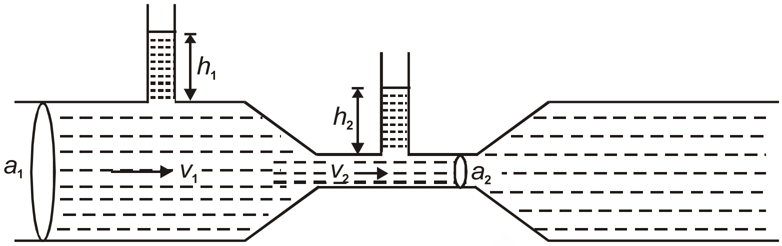
$$Q = a_1 a_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}}$$



Case - (b) :

Venturimeter

$$Q = a_1 a_2 \sqrt{\frac{2g(h_1 - h_2)}{a_1^2 - a_2^2}}$$

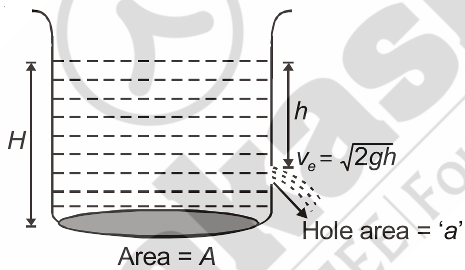


(2) Hole in a tank

(a) Speed of efflux $v_e = \sqrt{2gh}$ (If $a << A$). This is known as Torricelli's theorem.

If a is comparable to A then

$$v_e = \sqrt{2gh}\sqrt{\frac{A^2}{A^2 - a^2}}$$



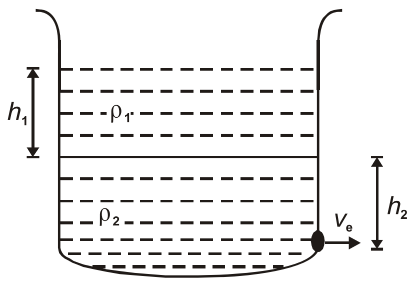
(b) Time taken by water level to fall from h_1 to h_2

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{h_1} - \sqrt{h_2}\right]$$

(c) Time taken to completely empty the container by a hole at bottom

$t \propto \sqrt{H}$ [Put $h_1 = H$, $h_2 = 0$]

(d) $v_e = \sqrt{2g\left(h_2 + \frac{\rho_1 h_1}{\rho_2}\right)}$ in the situation shown in figure

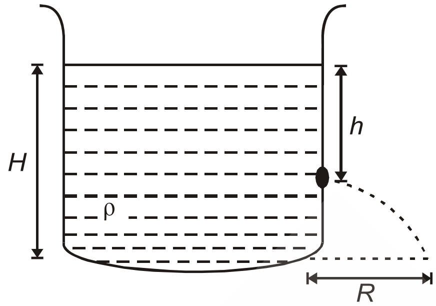


(e) Range of liquid

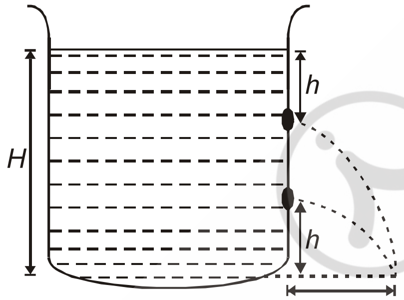
$R = 2\sqrt{h(H-h)}$

For a given value of total height (H)

$R_{max} = H$ when $h = \frac{H}{2}$



(f)

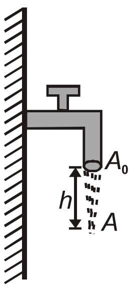


$R = 2\sqrt{h(H-h)}$ for both holes

(3) If A_0 = area of cross-section of mouth of tap

A = area of cross-section of water jet at a depth h

$A_0 v_0 = Av = Q$ [rate of flow]



By Bernoulli's theorem $\frac{v_0^2}{2} + gh = \frac{v_1^2}{2}$

[$\because$ pressure is atmospheric at both points]

Reynold's Number

$$N_R = \frac{\rho v D}{\eta} = \frac{\text{Inertial Force}}{\text{Viscous force}}$$

Value of N_R for various cases :

(1) $N_R < 2000$, flow is streamline

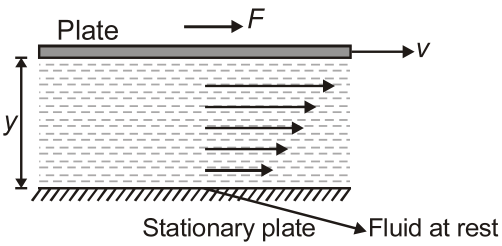
(2) $N_R > 3000$, flow is turbulent

(3) $2000 < N_R < 3000$, flow is unstable

(4) When $N_R = 2000$, flow is critical

$$\frac{\rho v D}{\eta} = 2000 \Rightarrow v_c = 2000\frac{\eta}{\rho D} \text{ (Critical velocity)}$$

Viscosity and Viscous Force



Viscous force is given in this case by,

$$F = -\eta A \frac{dv}{dy}$$

Units of η : SI → 1 Pa.s = 10 poise = 1 decapoise

C.G.S → 1 dyne/cm^2-s = 1 poise

Poiseuille's Equation

Volume flow rate across a tube with pressure difference between its ends is,

$$Q = \frac{dV}{dt} = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

Series combination of two tubes

Two tubes of radius r_1, length l_1 and radius r_2, length l_2 are connected in series across a pressure difference of P. Length of a single tube that can replace the two tubes is found using,

$$\frac{l}{r^4} = \frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}$$

STOKES LAW

When a small spherical body of radius r is moving with velocity v through a perfectly homogeneous medium having coefficient of viscosity η, it experiences a retarding force given by

$\boldsymbol{F = 6\pi\eta r v}$.

Important case :

(1) A body of radius r released from rest in a fluid

If σ = density of body

ρ = density of liquid or fluid

Terminal velocity is given by,

$$v_T = \frac{2}{9}\frac{r^2 g}{\eta}(\sigma - \rho)$$

Thermal Expansion

When the temperature of a body increases, its all dimensions (length, area, volume) increase.

(1) Coefficient of linear expansion is given by

$$\alpha = \frac{\Delta L}{L\Delta\theta}$$

$L_\theta = L_0\,(1 + \alpha\Delta\theta)$, $\Delta\theta$ is the change in temperature in °C or K & L_0 is initial length

(2) Coefficient of superficial expansion is given by

$$\beta = \frac{\Delta A}{A\Delta\theta}$$

$$A_\theta = A_0\,(1 + \beta\Delta\theta)$$

(3) Coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V\Delta\theta} \quad \text{or} \quad V_\theta = V_0\,(1 + \gamma\,\Delta\theta)$$

$$\frac{m}{\rho_\theta} = \frac{m}{\rho_0}(1+\gamma\,\Delta\theta) \quad \Rightarrow \quad \rho_\theta = \rho_0(1 - \gamma\Delta\theta)$$

An Isotropic body expands equally in all directions and we can obtain the following relations

$$\gamma = 3\alpha,\ \beta = 2\alpha \ \text{ or } \ \boxed{\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}},\ \gamma = \frac{3}{2}\beta$$

CALORIMETRY

(1) Specific heat capacity or Gram specific heat (c) : If Q heat is given to a substance of mass 'm', and rise in temperature is $\Delta\theta$, then

$$c = \frac{Q}{m\Delta\theta} \ (\text{cal/g°C})$$

(2) Molar heat capacity (C) :

C = Molar mass (M) × specific heat capacity

$$\Rightarrow \quad C = Mc = \frac{M.Q}{m\Delta\theta} = \frac{Q}{n\Delta\theta}$$

$$\boxed{C = \frac{Q}{n\Delta\theta}} : n = \frac{m}{M} \text{ is number of moles}$$

(3) Heat capacity of an object is defined as product of mass and specific heat.

(4) In general if Q heat is given to a substance of mass 'm' which increases its temperature by $\Delta\theta$ then

$\boxed{Q = mc\Delta\theta}$ c is specific heat capacity

or $\boxed{Q = nC\Delta\theta}$ C is molar heat capacity, n is number of moles of the substance.

Specific Latent heat

(1) Latent heat of fusion $L_f = \dfrac{Q}{m}$

(2) Latent heat of vaporisation $L_v = \dfrac{Q}{m}$

During phase change (liquid $\rightleftarrows$ solid or liquid $\rightleftarrows$ vapours) temperature remains constant, but internal energy changes.

Water

Specific heat C = 1 cal/gm/°C = 4.2 J/gm/°C = 4200 J/kg/°C

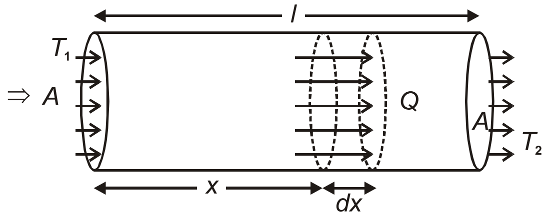
L_f = 80 cal/gm = 336 J/gm

L_v = 540 cal/gm = 2268 J/gm

For ice : C_{ice} = 0.5 cal/g°C = 2100 J/kg°C

Law of Conduction

Consider a rod of length l, cross sectional area A, with its ends maintained at temperatures T_1 & T_2 ($T_1 > T_2$). In steady state



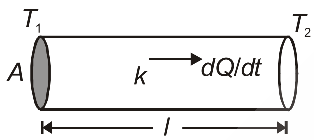
Rate of heat flow across any section is given by

$$H = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

Here k = Thermal conductivity and $\frac{dT}{dx}$ is known as temperature gradient i.e. rate of change of temperature with distance. k depends only on the nature of the material.

S.I. units of thermal conductivity is $Wm^{-1}K^{-1}$

THERMAL RESISTANCE OF A ROD



In steady state $\frac{dQ}{dt} = kA\frac{(T_1 - T_2)}{l}$

Thermal resistance, $R = \frac{l}{kA}$ $\left[\text{as in current electricity } R = \frac{\rho l}{A} = \frac{l}{\sigma A}\right]$

Wiedemann – Franz law

$\frac{k}{\sigma T}$ = constant $\Rightarrow$ a substance which is good conductor of heat (silver) is also a good conductor of electricity (mica & human body is exception to above law)

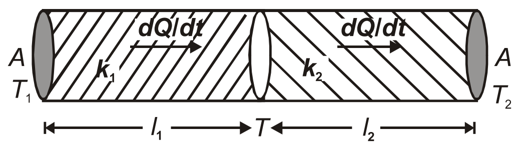
where σ is electrical conductivity

Composite Rod :

(1) Series

In steady state

$$R = \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} = \frac{l_1 + l_2}{kA}$$



Where k = effective thermal conductivity given by

For same area of cross section,

$$k = \frac{l_1 + l_2 + ... + l_n}{\frac{l_1}{k_1} + \frac{l_2}{k_2} + ... + \frac{l_n}{k_n}}$$

For two slabs of equal length

$$k = \frac{2k_1k_2}{k_1 + k_2}$$

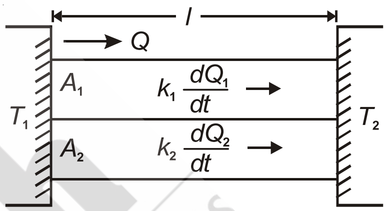
Temperature of junction

$$T = \frac{\frac{k_1}{l_1}T_1 + \frac{k_2}{l_2}T_2}{\frac{k_1}{l_1} + \frac{k_2}{l_2}}.$$

For same geometrical dimensions,

$$T = \frac{k_1T_1 + k_2T_2}{k_1 + k_2}$$

(2) In parallel



$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \frac{k(A_1 + A_2)}{l} = \frac{k_1A_1}{l} + \frac{k_2A_2}{l}$$

where k = effective coefficient of thermal conductivity given by

$$k = \frac{k_1A_1 + k_2A_2}{A_1 + A_2}$$

Example, for two slabs of equal area $k = \frac{k_1 + k_2}{2}$

STEFAN'S LAW

The radiant energy emitted by a perfectly black body per second per unit area (emissive power) is directly proportional to the fourth power of the absolute temperature of the body.

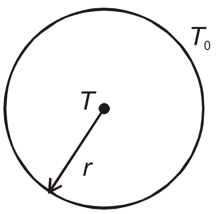
$R \propto T^4$ $\qquad$ $R = \sigma T^4$

$R = \frac{\text{Power}}{\text{Area}} \Rightarrow P = A\sigma T^4$ $\qquad$ ($\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4})

For other bodies $P = \varepsilon A\sigma T^4$, ε is emissivity of the body.

Rate of heat loss

For a sphere of radius r at a temperature T placed in a surrounding of temperature T_0, the rate of heat loss is $\frac{dQ}{dt} = 4\pi r^2\varepsilon\sigma(T^4 - T_0^4)$, where ε is emissivity.



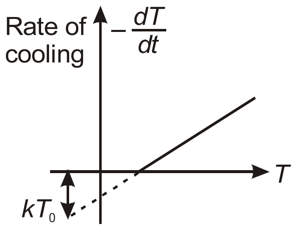
Rate of cooling

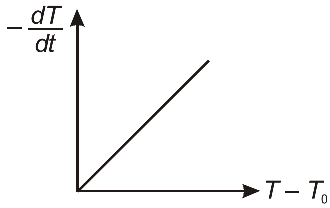
For a sphere of radius r, density ρ and specific heat capacity s.

The rate of fall in temperature is given by $\frac{dT}{dt} = -\frac{3\varepsilon\sigma(T^4 - T_0^4)}{\rho s r}$

Newton's Law of Cooling

If the temperature T of a body is not much different from surrounding temperature T_0, then rate of cooling of a liquid is directly proportional to the difference in the temperature of liquid T and temperature of surroundings (T_0) *i.e.*





Rate of cooling $= \left(-\frac{dT}{dt}\right) \propto (T - T_0)$

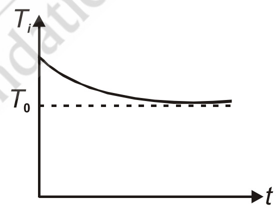
$$-\frac{dT}{dt} = \alpha(T - T_0)$$

Results

(1) $T_f = T_0 + (T_i - T_0)e^{-\alpha t}$, where T_i is initial temperature, T_f is temperature after time t.

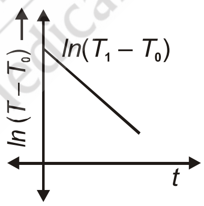
(2) Another form $\alpha t = \log\left|\frac{T_i - T_0}{T_f - T_0}\right|$

(3) $\frac{-dT}{dt} = \frac{4\varepsilon A\sigma T_0^3}{mc}(T - T_0)$, $\begin{bmatrix} m = \text{mass of body} \\ c = \text{specific heat} \\ A = \text{surface area} \\ \varepsilon = \text{emissivity} \end{bmatrix}$



(4) Another approximate formula is

$$\frac{T_1 - T_2}{t} = \alpha\left(\frac{T_1 + T_2}{2} - T_0\right)$$



Above formula gives time 't' taken by the body to cool down from T_1 to T_2. T_0 is temperature of surrounding.

(5) If temperature of a body changes from θ_1 to θ_2 in time 't' and changes from θ_2 to θ_3 in next time then

$$\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = \frac{\theta_3 - \theta_0}{\theta_2 - \theta_0} \quad (\theta_0 = \text{temperature of environment})$$

(6) If equal masses of two liquids having same surface area and finish, cool from same initial temperature to same final temperature with same surrounding, then

$$\frac{t_1}{t_2} = \frac{k_2}{k_1} = \frac{c_1}{c_2}\,; \qquad c_1 \,\&\, c_2 \text{ are specific heats}$$

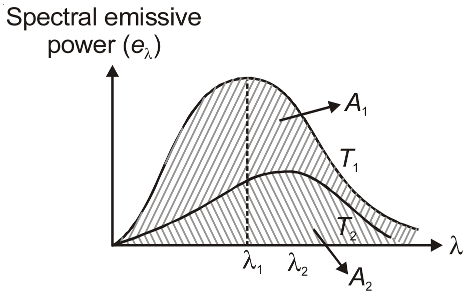
WIEN'S DISPLACEMENT LAW

This law states that the wavelength corresponding to maximum intensity for a black body is inversely proportional to the absolute temperature of the body

$$\lambda_m = \frac{b}{T}$$

where b is a constant known as Wien's constant

Results



$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}} \quad \boxed{\frac{A_1}{A_2} = \frac{T_1^4}{T_2^4}}$$

(1) $\lambda_{max} T = b$

(2) $b = 2.898 \times 10^{-3}$ m-K

(3) Area under $e_\lambda - \lambda$ graph $= \sigma T^4$ (Total emissive power)

(4) If the temperature of the black body is made two fold, λ_{max} becomes half, while area becomes 16 times.

(5) Temperature of the Sun,

If T = temperature of sun, then total energy radiated by sun per second $= \sigma T^4 (4\pi R^2)$

Intensity at distance r from the sun (*i.e.*, on earth)

$$I = \text{(Power radiated)(Area)} = \frac{\sigma T^4 R^2}{r^2} = S$$, where S is called solar constant [$S = 1.4$ kW / m^2]

$$\text{So, } T = \left[\left(\frac{r^2}{R^2}\right)\frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5\times10^8}{7\times10^5}\right)^2 \times \frac{1.4\times10^3}{5.67\times10^{-8}}\right]^{\frac{1}{4}} \approx 5800 \text{ K}$$

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