Chapter 4

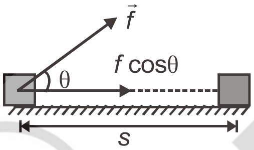
# Work, Energy and Power

## CONCEPT OF WORK

$W = \vec{f}.\vec{s} \quad (\vec{s} = \vec{s}_2 - \vec{s}_1)$

$= fs\cos\theta$

$= s\,(f\cos\theta)$




## KINETIC ENERGY

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

## WORK-ENERGY THEOREM

$$W_{\text{Total}} = \Delta\text{K.E.}$$

or

$$W_{\text{ext.}} + W_{\text{int.}} = \Delta\text{K.E}$$

## Work Done by Spring Force

$$W = -\int_{x_1}^{x_2} kx\,dx = \frac{-1}{2}k(x_2^2 - x_1^2)$$

when $x_2 > x_1$ , $W < 0$

when $x_2 < x_1$ , $W > 0$

## COLLISION

### One dimension

$$\overset{u_1}{\rightarrow}\ (m_1)\quad \overset{u_2}{\rightarrow}\ (m_2) \quad \Rightarrow \quad \overset{v_1}{\rightarrow}\ (m_1)\quad \overset{v_2}{\rightarrow}\ (m_2) \qquad (u_1 > u_2 \text{ and } v_2 > v_1)$$

$$\Delta\text{KE} = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2(1 - e^2)$$

Following are the important cases

1. **Elastic collision $e$ = 1**

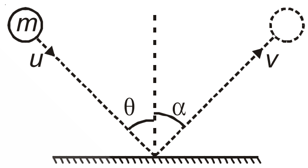
$$v_2 - v_1 = u_1 - u_2$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{m_2 - m_1}{m_2 + m_1}u_2 + \frac{2m_1u_1}{m_2 + m_1}$$

$\Delta$KE = 0 $\Rightarrow$ Final KE = Initial KE

2. **Coefficient of restitution = $e$**



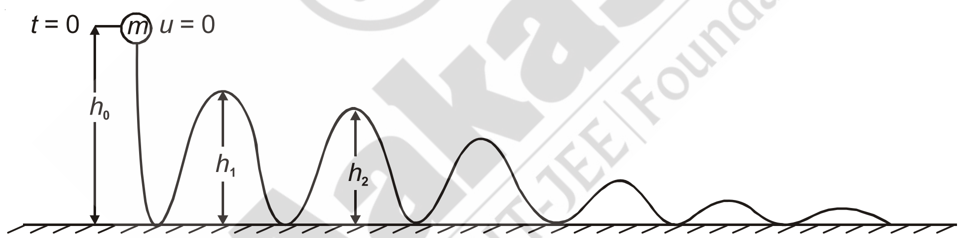

$u \sin\theta = v \sin\alpha$ ...(1)

$eu \cos\theta = v \cos\alpha$ ...(2)

$\Rightarrow$ $v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

$\tan\alpha = \dfrac{\tan\theta}{e}$ ($i.e.$ $\alpha > \theta$)

3. **A ball of mass $m$ is dropped from a height $h_0$ on an inelastic floor.**




The coefficient of restitution = $e$

(a) Maximum height after $n^{th}$ bounce is $h_n = e^{2n}h_0$

(b) Speed of rebound after $n^{th}$ bounce = $e^n\sqrt{2gh_0}$

(c) Total distance travelled before the body comes to rest

$$= h_0\left[\frac{1+e^2}{1-e^2}\right]$$

(d) The time after which the body comes to rest

$$= \sqrt{\frac{2h_0}{g}}\cdot\left[\frac{1+e}{1-e}\right]$$

(e) Average force exerted on the ground is $mg$

(f) Displacement of ball when it stops is $h_0$

**4. Oblique elastic collision**

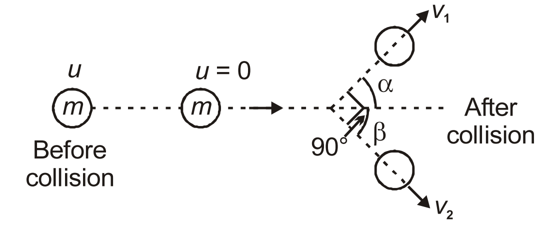
A body of mass $m$ collides with a stationary body of same mass.

(a) $\alpha + \beta = \dfrac{\pi}{2}$ ...(1)

(b) $v_1\cos\alpha + v_2\cos\beta = u$ ...(2)

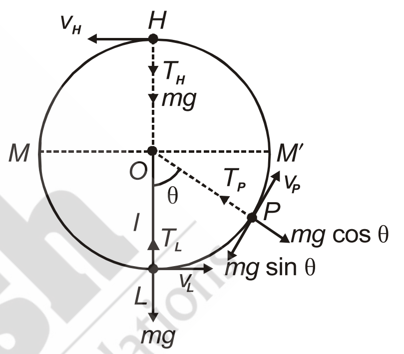
(c) $v_1\sin\alpha = v_2\sin\beta$ ...(3)

(d) $u^2 = v_1^2 + v_2^2$ ...(4)




## MOTION IN A VERTICAL CIRCLE

A particle of mass $m$ is tied to a string of length $l$ whose other end is fixed. The particle can revolve about $O$ in a vertical circle. when it is at position $L$ (lowest point), it is given a speed $V_L$ horizontal. Following results are useful in describing its motion.




1. $a_T = g\sin\theta$ ...(1)

2. $a_C = \dfrac{T_p - mg\cos\theta}{m} = \dfrac{v_p^2}{l}$

3. $T_p = \dfrac{mv_p^2}{l} + mg\cos\theta$ ...(2)

4. $T_L = mg + \dfrac{mv_L^2}{l}$

5. $v_L \le \sqrt{2gl}$ , it oscillates between $M$ and $M'$

6. $\sqrt{2gl} < v_L < \sqrt{5gl}$ , it will leave the circular path somewhere between $M'$ and $H$.

7. When $v_L \ge \sqrt{5gl}$ , it completes vertical circle (Also $v_H \ge \sqrt{gl}$ )

8. $T_H = -mg + \dfrac{mv_H^2}{l}$

9. $T_L - T_H = 6\,mg$ (always)

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